Assignment 8
Semantics, WS 2011-2012

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Read in the lecture notes:

Exercise 8.1 For sequential Imp, the small-step semantics agrees with the big-step semantics. The file *agreement.v* contains all necessary definitions and lemmas for the agreement proof with some of the proofs deleted. Fill in the missing proofs.

Exercise 8.2 Prove the following goals.

a) Goal forall r, reflexive r -> ~ exists x, normal r x.

b) Goal forall r s, functional r -> functional s -> functional (comp r s).

c) Do not use *firstorder* or *eauto*.

Goal forall r s, reflexive r -> reflexive s -> reflexive (comp r s).

d) Do not use *firstorder*.

Lemma transitive_rap r s :
rap r s -> transitive s -> rap (comp r s) s.

e) Do not use *firstorder*. Hint: Use *hnf* as a convenient means for top-level unfolding.

Lemma reflexive_rap r s :
rap r s -> reflexive s -> rap r (comp r s).

Exercise 8.3 Prove the following goals.

a) **Lemma** star_expansive r :
rap r (star r).

b) **Lemma** normal_form_functional (r : rel) :
functional r -> functional (normal_form r).

c) **Lemma** star_least r s :
reflexive s -> transitive s -> rap r s -> rap (star r) s.

d) **Lemma** star_idempotent r :
req (star (star r)) (star r).

Exercise 8.4 Prove the lemma used to prove that the diamond property implies confluence.

```
Lemma diamond_confluence' (r : rel) x y z :
diamond r -> r x y -> star r x z -> joinable (star r) y z.
```

Exercise 8.5 Prove the following goals.

```
Lemma joinable_sym (r : rel) x y :
joinable r x y -> joinable r y x.

Lemma joinable_1 (r : rel) x y z :
r x y -> joinable (star r) y z -> joinable (star r) x z.

Lemma joinable_star (r : rel) x y z :
star r x y -> joinable (star r) y z -> joinable (star r) x z.
```

Exercise 8.6 A relation r is **locally confluent** if y and z are joinable by r^* whenever rxy and rxz. In Coq, the definition is given as follows.

```
Definition locally_confluent (r : rel) : Prop :=
forall x y z, r x y -> r x z -> joinable (star r) y z.
```

Give an example of a relation (on a finite X) that is locally confluent, but not confluent.