Semantics, WS 2011-2012: Solution for Assignment 8

Prof. Dr. Gert Smolka, Dr. Chad Brown

Exercise 8.1 For sequential Imp, the small-step semantics agrees with the big-step semantics. The file *agreement.v* contains all necessary definitions and lemmas for the agreement proof with some of the proofs deleted. Fill in the missing proofs.

Solution to Exercise 8.1 See the Coq file.

Exercise 8.2 Prove the following goals.

a) Goal forall r, reflexive r -> ~ exists x, normal r x.

b) Goal forall r s, functional r -> functional s -> functional (comp r s).

c) Do not use *firstorder* or *eauto*.

Goal forall r s, reflexive r -> reflexive s -> reflexive (comp r s).

d) Do not use *firstorder*.

Lemma transitive_rap r s :
rap r s -> transitive s -> rap (comp r s) s.

e) Do not use *firstorder*. Hint: Use *hnf* as a convenient means for top-level unfolding.

Lemma reflexive_rap r s :
rap r s -> reflexive s -> rap r (comp r s).

Solution to Exercise 8.2 See the Coq file.

Exercise 8.3 Prove the following goals.

a) Lemma star_expansive r :
rap r (star r).

b) Lemma normal_form_functional (r : rel) :
functional r -> functional (normal_form r).

c) Lemma star_least r s :
reflexive s -> transitive s -> rap r s -> rap (star r) s.

d) **Lemma** star_idempotent r :
req (star (star r)) (star r).

Solution to Exercise 8.3 See the Coq file.

Exercise 8.4 Prove the lemma used to prove that the diamond property implies confluence.

Lemma diamond_confluence' (r : rel) x y z :
diamond r -> r x y -> star r x z -> joinable (star r) y z.

Solution to Exercise 8.4 See the Coq file.

Exercise 8.5 Prove the following goals.

Lemma joinable_sym (r : rel) x y :
joinable r x y -> joinable r y x.

Lemma joinable_1 (r : rel) x y z :
r x y -> joinable (star r) y z -> joinable (star r) x z.

Lemma joinable_star (r : rel) x y z :
star r x y -> joinable (star r) y z -> joinable (star r) x z.

Solution to Exercise 8.5 See the Coq file.

Exercise 8.6 A relation r is **locally confluent** if y and z are joinable by r^* whenever rxy and rxz. In Coq, the definition is given as follows.

Definition locally_confluent (r : rel) : Prop :=
forall x y z, r x y -> r x z -> joinable (star r) y z.

Give an example of a relation (on a finite X) that is locally confluent, but not confluent.

Solution to Exercise 8.6 $0 \leftarrow 1 \leftrightarrow 2 \rightarrow 3$