Semantics, WS 2011-2012: Solution for Assignment 11

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Exercise 11.1 Write a function in T that adds two numbers. Translate your function to Coq and test it for some arguments. Note that T translates directly to Coq with *primrec* as defined above.

Solution to Exercise 11.1

```
Definition T_add : nat -> nat -> nat :=
fun n m:nat => primrec n m (fun _ r => (S r)).

Compute T_add 2 3.
Compute T_add 4 2.
```

Exercise 11.2 Define an abstract syntax and a small-step semantics for T in Coq. Follow the development of PCF.

Solution to Exercise 11.2 See the Coq file.

Exercises 11.3 - 11.7 are about the functional language E.

Exercise 11.3

a) Normal terms that are not values are called **stuck**. Find a stuck term.

b) Find an example showing that the step relation does not preserve types from right to left.

Solution to Exercise 11.3

a) $S\ false$.

b) $if\ true\ true\ O$

Exercise 11.4 Suppose we add two new reduction rules:

$$P\ true \to P\ false$$
$$P\ false \to P\ true$$

Which of the following properties remain true in the presence of these rules?

a) Determinacy of *step*

b) Termination of *step* for well-typed terms

c) Progress

d) Preservation

Solution to Exercise 11.4 All the properties remain true.

Exercise 11.5 Suppose we add a new typing rule:

$$\frac{t_1 : T}{\mathit{if}\ \mathit{true}\ t_1\ t_2 : T}$$

Which of the following properties remain true in the presence of these rules?

a) Determinacy of *step*

b) Termination of *step* for well-typed terms

c) Progress

d) Preservation

Solution to Exercise 11.5 All the properties remain true.

Exercise 11.6 Prove the following lemmas.

```
Lemma value_normal t t' :
value t -> step t t' -> False.

Lemma preservation t T t' :
type t T -> step t t' -> type t' T.

Lemma progress t T :
type t T -> value t \/ exists t', step t t'.

Lemma type_unique t T T' :
type t T -> type t T' -> T = T'.

Lemma step_deterministic t t1 t2 :
step t t1 -> step t t2 -> t1 = t2.
```

Solution to Exercise 11.6 See the Coq file.

Exercise 11.7 Prove that *step* terminates.

Solution to Exercise 11.7 Each step reduces the size of the term. See the Coq file.

Exercise 11.8 Prove the following lemmas about the type-indexed version of E.

```
Lemma step_deterministic (T : ty) (t t1 t2 : tm T) :
step t t1 -> step t t2 -> t1 = t2.

Lemma Progress (T : ty) (t : tm T) :
value t \/ exists t', step t t'.
```

Solution to Exercise 11.8 See the Coq file.

Exercise 11.9 Formalize STLC in Coq. For the abstract syntax and the small-step semantics of follow the development of PCF. For contexts and the typing relation follow the development in the SF text.

Solution to Exercise 11.9 See the Coq file from the class of January 23.

Exercise 11.10 (Optional) Reconsider the functional language E. Write a function $tycheck : tm \to option\ ty$ and prove the following lemma.

```
Lemma tycheck_correct t T :
type t T <-> tycheck t = Some T.
```

Solution to Exercise 11.10 (Optional) See the Coq file.