Assignment 8
Semantics, WS 2013/14

Prof. Dr. Gert Smolka, Steven Schäfer
www.ps.uni-saarland.de/courses/sem-ws13/

Read in the lecture notes: Chapter 2 & 3

Exercise 8.1 Abstraction in CL.

a) Show that for all x, s, the abstraction ${}^{x}s$ is normal.

b) Soundness of abstraction: $({}^{x}s)t \equiv s_t^x$.

Exercise 8.2 In λ-calculus we have $WN\ s \to WN(sx)$ for all terms s and variables x. Show that this is not the case in CL (on paper).

Exercise 8.3

$$\begin{aligned} I &:= SKK \\ B &:= S(KS)K \\ \omega &:= SII \\ A &:= B(SI)\omega \\ T &:= AA \end{aligned}$$

Show the following equivalences.

a) $Is \equiv s$

b) $Bstu \equiv s(tu)$

c) $\omega s \equiv ss$

d) $Ast \equiv t(sst)$

e) $Ts \equiv s(Ts)$

Exercise 8.4 We define the Church numerals in CL as follows:

$$\begin{aligned} zero &:= KI \\ succ &:= SB \\ add &:= SI(K succ) \\ N\ n &:= succ^n\ zero \end{aligned}$$

Show the correctness of this definition.

a) Prove that $N\ n$ is normal for all n.

b) $add\ zero\ s \equiv s$

c) $add(succ\ s)t \equiv succ(add\ st)$

d) $add(N\ n)(N\ m) \equiv N(n+m)$

e) $add(N\ n)(N\ m) \succeq^* N(n+m)$

f) Show that N is injective.

Exercise 8.5 (Recursive abstraction) Define a function $\mu : nat \to term \to term$ such that $\mu\ x\ s \equiv s^x_{\mu\ x\ s}$.