Appendix for Call-By-Push-Value in Coq: Operational, Equational, and Denotational Theory

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A Figures

Parallel reduction $M \rightsquigarrow M'$ *and* $V \rightsquigarrow V'$

$$x \rightsquigarrow x \qquad () \rightsquigarrow () \qquad \frac{V_1 \rightsquigarrow V_1' \quad V_2 \rightsquigarrow V_2'}{(V_1, V_2) \rightsquigarrow (V_1', V_2')} \qquad \frac{V \rightsquigarrow V'}{\mathbf{inj}_i\ V \rightsquigarrow \mathbf{inj}_i\ V'}$$

$$\frac{M \rightsquigarrow M'}{\{M\} \rightsquigarrow \{M'\}} \qquad \langle\rangle \rightsquigarrow \langle\rangle \qquad \frac{V \rightsquigarrow V'}{V! \rightsquigarrow V'!} \qquad \frac{M \rightsquigarrow M'}{\{M\}! \rightsquigarrow M'}$$

$$\frac{M \rightsquigarrow M'}{\lambda x.M \rightsquigarrow \lambda x.M'} \qquad \frac{M \rightsquigarrow M' \quad V \rightsquigarrow V'}{M\ V \rightsquigarrow M'\ V'}$$

$$\frac{M \rightsquigarrow M' \quad V \rightsquigarrow V'}{(\lambda x.M)\ V \rightsquigarrow M'[V'/x]}$$

$$\frac{M_1 \rightsquigarrow M_1' \quad M_2 \rightsquigarrow M_2'}{\langle M_1, M_2\rangle \rightsquigarrow \langle M_1', M_2'\rangle} \qquad \frac{V \rightsquigarrow V'}{\mathbf{return}\ V \rightsquigarrow \mathbf{return}\ V'}$$

$$\frac{M \rightsquigarrow M' \quad N \rightsquigarrow N'}{\mathbf{let}\ x \leftarrow M\ \mathbf{in}\ N \rightsquigarrow \mathbf{let}\ x \leftarrow M'\ \mathbf{in}\ N'}$$

$$\frac{V \rightsquigarrow V' \quad N \rightsquigarrow N'}{\mathbf{let}\ x \leftarrow \mathbf{return}\ V\ \mathbf{in}\ N \rightsquigarrow N'[V'/x]}$$

$$\frac{M \rightsquigarrow M'}{\mathbf{prj}_i\ M \rightsquigarrow \mathbf{prj}_i\ M'} \qquad \frac{M_1 \rightsquigarrow M_1' \quad M_2 \rightsquigarrow M_2'}{\mathbf{prj}_i\ \langle M_1, M_2\rangle \rightsquigarrow M_i'}$$

$$\frac{V \rightsquigarrow V'}{\mathbf{case_0}(V) \rightsquigarrow \mathbf{case_0}(V')}$$

$$\frac{V \rightsquigarrow V' \quad M_1 \rightsquigarrow M_1' \quad M_2 \rightsquigarrow M_2'}{\mathbf{case}(V, x_1.M_1, x_2.M_2) \rightsquigarrow \mathbf{case}(V', x_1.M_1', x_2.M_2')}$$

$$\frac{V \rightsquigarrow V' \quad M_1 \rightsquigarrow M_1' \quad M_2 \rightsquigarrow M_2'}{\mathbf{case}(\mathbf{inj}_i\ V, x_1.M_1, x_2.M_2) \rightsquigarrow M_i'[V'/x_i]}$$

$$\frac{V \rightsquigarrow V' \quad M \rightsquigarrow M'}{\mathbf{split}(V, x.y.M) \rightsquigarrow \mathbf{split}(V', x.y.M')}$$

$$\frac{V_1 \rightsquigarrow V_1' \quad V_2 \rightsquigarrow V_2' \quad M \rightsquigarrow M'}{\mathbf{split}((V_1, V_2), x.y.M) \rightsquigarrow M'[V_1'/x, V_2'/y]}$$

Figure 1. CBPV parallel reduction

Parallel Reduction Function $\varrho\ M$ *and* $\varrho\ V$

$$\varrho\ x := x \qquad \varrho\ () := () \qquad \varrho\ (V_1, V_2) := (\varrho\ V_1, \varrho\ V_2)$$

$$\varrho\ (\mathbf{inj}_i\ V) := \mathbf{inj}_i\ (\varrho\ V)$$

$$\varrho\ \{M\} := \{\varrho\ M\} \qquad \varrho\ \langle\rangle := \langle\rangle \qquad \varrho\ (\{M\}!) := \varrho\ M$$

$$\varrho\ (V!) := (\varrho\ V)! \quad \text{where } V \text{ is not a thunk}$$

$$\varrho\ (\lambda x.M) := \lambda x.\varrho\ M \qquad \varrho\ ((\lambda x.M)\ V) := (\varrho\ M)[\varrho\ V/x]$$

$$\varrho\ (M\ V) := (\varrho\ M)\ (\varrho\ V) \quad \text{where } M \text{ is not a lambda}$$

$$\varrho\ \langle M_1, M_2\rangle := \langle \varrho\ M_1, \varrho\ M_2\rangle \qquad \varrho\ (\mathbf{return}\ V) := \mathbf{return}\ \varrho\ V$$

$$\varrho\ (\mathbf{let}\ x \leftarrow \mathbf{return}\ V\ \mathbf{in}\ N) := (\varrho\ N)[\varrho\ V/x]$$

$$\varrho\ (\mathbf{let}\ x \leftarrow M\ \mathbf{in}\ N) := \mathbf{let}\ x \leftarrow \varrho\ M\ \mathbf{in}\ \varrho\ N \quad \text{where } M \text{ is not a return}$$

$$\varrho\ (\mathbf{prj}_i\ \langle M_1, M_2\rangle) := \varrho\ M_i$$

$$\varrho\ (\mathbf{prj}_i\ M) := \mathbf{prj}_i\ (\varrho\ M) \quad \text{where } M \text{ is not a pair}$$

$$\varrho\ (\mathbf{case_0}(V)) := \mathbf{case_0}(\varrho\ V)$$

$$\varrho\ (\mathbf{case}(\mathbf{inj}_i\ V, x_1.M_1, x_2.M_2)) := (\varrho\ M_i)[\varrho\ V/x_i]$$

$$\varrho\ (\mathbf{case}(V, x_1.M_1, x_2.M_2)) :=$$
$$\mathbf{case}(\varrho\ V, x_1.\varrho\ M_1, x_2.\varrho\ M_2) \quad \text{where } V \text{ is not a injection}$$

$$\varrho\ (\mathbf{split}((V_1, V_2), x.y.M)) := (\varrho\ M)[\varrho\ V_1/x, \varrho\ V_2/y]$$

$$\varrho\ (\mathbf{split}(V, x.y.M)) := \mathbf{split}(\varrho\ V, x.y.\varrho\ M) \quad \text{where } V \text{ is not a pair}$$

Figure 2. CBPV reduction function

CBN translation relation $\boxed{s \mapsto_n M}$

$$\frac{}{x \mapsto_n x!} \qquad \frac{}{() \mapsto_n \mathbf{return}\ ()} \qquad \frac{s_1 \mapsto_n M_1 \quad s_2 \mapsto_n M_2}{(s_1, s_2) \mapsto_n \langle M_1, M_2\rangle}$$

$$\frac{s \mapsto_n M}{\mathbf{inj}_i\ s \mapsto_n \mathbf{return}\ \mathbf{inj}_i\ \{M\}}$$

$$\frac{s \mapsto_n M \quad t_1 \mapsto_n N_1 \quad t_2 \mapsto_n N_2}{\mathbf{case}(s, x_1.t_1, x_2.t_2) \mapsto_n \mathbf{let}\ y \Leftarrow M\ \mathbf{in}\ \mathbf{case}(y, x_1.N_1, x_2.N_2)}$$

$$\frac{s \mapsto_n M}{\mathbf{prj}_i\ s \mapsto_n \mathbf{prj}_i\ M} \qquad \frac{s \mapsto_n M}{\lambda x.s \mapsto_n \lambda x.M} \qquad \frac{s \mapsto_n M \quad t \mapsto_n N}{s\ t \mapsto_n M\ \{N\}}$$

Figure 3. CBN translation relation to CBPV

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