

`one` is a constant of type `term`  
`ap` is a constant of type `term → term → term`  
`lam` is a constant of type `term → term`  
`sub` is a constant of type `term → subst → term`  
`id` is a constant of type `subst`  
`sh` is a constant of type `subst`  
`push` is a constant of type `term → subst → subst`  
`comp` is a constant of type `subst → subst → subst`  
`axapp` is an abbreviation defined by

$$\forall A_{\text{term}} B_{\text{term}} M_{\text{subst}}. \text{sub}(\text{ap } A B) M = \text{ap}(\text{sub } A M)(\text{sub } B M)$$

*Same formula in array format:*

$$\forall A_{\text{term}} B_{\text{term}} M_{\text{subst}}. \text{sub}(\text{ap } A B) M = \text{ap}(\text{sub } A M)(\text{sub } B M)$$

`axvarcons` is an abbreviation defined by

$$\forall A_{\text{term}} M_{\text{subst}}. \text{sub one}(\text{push } A M) = A$$

`axvarid` is an abbreviation defined by

$$\forall A_{\text{term}}. \text{sub } A \text{ id} = A$$

`axabs` is an abbreviation defined by

$$\forall A_{\text{term}} M_{\text{subst}}. \text{sub}(\text{lam } A) M = \text{lam}(\text{sub } A(\text{push one}(\text{comp } M \text{ sh})))$$

`axclos` is an abbreviation defined by

$$\forall A_{\text{term}} M_{\text{subst}} N_{\text{subst}}. \text{sub}(\text{sub } A M) N = \text{sub } A(\text{comp } M N)$$

`axidl` is an abbreviation defined by

$$\forall M_{\text{subst}}. \text{comp id } M = M$$

`axshiftcons` is an abbreviation defined by

$$\forall A_{\text{term}} M_{\text{subst}}. \text{comp sh}(\text{push } A M) = M$$

`axassoc` is an abbreviation defined by

$$\forall M_{\text{subst}} N_{\text{subst}} K_{\text{subst}}. \text{comp}(\text{comp } M N) K = \text{comp } M(\text{comp } N K)$$

*Same formula in array format:*

$$\forall M_{\text{subst}} N_{\text{subst}} K_{\text{subst}}. \text{comp}(\text{comp } M N) K = \text{comp } M(\text{comp } N K)$$

**axmap** is an abbreviation defined by

$$\forall A_{\text{term}} M_{\text{subst}} N_{\text{subst}}. \text{comp} (\text{push } A \ M) \ N = \text{push} (\text{sub } A \ N) (\text{comp } M \ N)$$

*Same formula in array format:*

$$\forall A_{\text{term}} M_{\text{subst}} N_{\text{subst}}. \text{comp} (\text{push } A \ M) \ N = \text{push} (\text{sub } A \ N) (\text{comp } M \ N)$$

**axidr** is an abbreviation defined by

$$\forall M_{\text{subst}}. \text{comp } M \ \text{id} = M$$

**axvarshift** is an abbreviation defined by

$$\text{push one sh} = \text{id}$$

**axscons** is an abbreviation defined by

$$\forall M_{\text{subst}}. \text{push} (\text{sub one } M) (\text{comp sh } M) = M$$

**var** is a constant of type  $\text{term} \rightarrow o$

**ulamvar1** is an abbreviation defined by

$$\text{var one}$$

**ulamvarsh** is an abbreviation defined by

$$\forall A_{\text{term}}. \text{var } A \Rightarrow \text{var} (\text{sub } A \ \text{sh})$$

**ulamvarind** is an abbreviation defined by

$$\forall P_{\text{term} \rightarrow o}. P \ \text{one} \Rightarrow (\forall A_{\text{term}}. \text{var } A \Rightarrow P \ A \Rightarrow P (\text{sub } A \ \text{sh})) \Rightarrow \forall A_{\text{term}}. \text{var } A \Rightarrow P \ A$$

*Same formula in array format:*

$$\forall P_{\text{term} \rightarrow o}. P \ \text{one} \Rightarrow (\forall A_{\text{term}}. \text{var } A \Rightarrow P \ A \Rightarrow P (\text{sub } A \ \text{sh})) \Rightarrow \forall A_{\text{term}}. \text{var } A \Rightarrow P \ A$$

**apinj1** is an abbreviation defined by

$$\forall A_{\text{term}} B_{\text{term}} C_{\text{term}} D_{\text{term}}. \text{ap } A \ C = \text{ap } B \ D \Rightarrow A = B$$

**apinj2** is an abbreviation defined by

$$\forall A_{\text{term}} B_{\text{term}} C_{\text{term}} D_{\text{term}}. \text{ap } A \ C = \text{ap } B \ D \Rightarrow C = D$$

**laminj** is an abbreviation defined by

$$\forall A_{\text{term}} B_{\text{term}}. \text{lam } A = \text{lam } B \Rightarrow A = B$$

`shinj` is an abbreviation defined by

$$\forall A_{\text{term}} B_{\text{term}}. \text{sub } A \text{ sh} = \text{sub } B \text{ sh} \Rightarrow A = B$$

`lamnotap` is an abbreviation defined by

$$\forall A_{\text{term}} B_{\text{term}} C_{\text{term}}. \text{lam } A \neq \text{ap } B C$$

`apnotvar` is an abbreviation defined by

$$\forall A_{\text{term}} B_{\text{term}}. \neg \text{var} (\text{ap } A B)$$

`lamnotvar` is an abbreviation defined by

$$\forall A_{\text{term}}. \neg \text{var} (\text{lam } A)$$

`induction` is an abbreviation defined by

$$\forall P_{\text{term} \rightarrow o}. (\forall A_{\text{term}}. \text{var } A \Rightarrow P A) \Rightarrow (\forall A_{\text{term}} B_{\text{term}}. P A \Rightarrow P B \Rightarrow P (\text{ap } A B)) \Rightarrow (\forall A_{\text{term}}. P A \Rightarrow P (\text{lam } A)) \Rightarrow \forall$$

*Same formula in array format:*

$$\begin{aligned} & \forall P_{\text{term} \rightarrow o}. (\forall A_{\text{term}}. \text{var } A \Rightarrow P A) \Rightarrow (\forall A_{\text{term}} B_{\text{term}}. P A \Rightarrow \\ & P B \Rightarrow P (\text{ap } A B)) \Rightarrow (\forall A_{\text{term}}. P A \Rightarrow P (\text{lam } A)) \Rightarrow \forall A_{\text{term}}. P A \end{aligned}$$

`pushprop_p_and_p_prime` is an abbreviation defined by

$$\lambda A_{\text{term}} M_{\text{subst}} P_{\text{term} \rightarrow o} Q_{\text{term} \rightarrow o}. \forall X_{\text{term}}. Q X \Leftrightarrow P (\text{sub } X (\text{push } A M))$$

*Same formula in array format:*

$$\begin{aligned} & \lambda A_{\text{term}} M_{\text{subst}} P_{\text{term} \rightarrow o} Q_{\text{term} \rightarrow o}. \forall X_{\text{term}}. Q X \\ & \Leftrightarrow P (\text{sub } X (\text{push } A M)) \end{aligned}$$

`pushprop_lem0` is an abbreviation defined by

$$\forall P_{\text{term} \rightarrow o} A_{\text{term}} M_{\text{subst}}. \exists Q_{\text{term} \rightarrow o}. \text{pushprop\_p\_and\_p\_prime } A M P Q$$

*Same formula in array format:*

$$\begin{aligned} & \forall P_{\text{term} \rightarrow o} A_{\text{term}} M_{\text{subst}}. \exists Q_{\text{term} \rightarrow o}. \text{pushprop\_p\_and\_p\_prime} \\ & A M P Q \end{aligned}$$

`pushprop_lem0_gthm` is an abbreviation defined by

$$\text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \Rightarrow \text{axmap} \Rightarrow \text{axidr}$$

Same formula in array format:

$$\begin{aligned} & \text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \\ & \Rightarrow \text{axmap} \Rightarrow \text{axidr} \Rightarrow \text{axvarshift} \Rightarrow \text{axscons} \Rightarrow \text{ulamvar1} \Rightarrow \text{ulamvarsh} \Rightarrow \text{ulamvarind} \\ & \Rightarrow \text{apinj1} \Rightarrow \text{apinj2} \Rightarrow \text{laminj} \Rightarrow \text{shinj} \Rightarrow \text{lamnotap} \Rightarrow \text{apnotvar} \Rightarrow \text{lamnotvar} \Rightarrow \text{induction} \\ & \Rightarrow \text{pushprop\_lem0} \end{aligned}$$

pushprop\_lem0.lthm is an abbreviation defined by

$$\text{pushprop\_lem0}$$

pushprop\_lem1v2 is an abbreviation defined by

$$\forall P_{\text{term} \rightarrow o} Q_{\text{term} \rightarrow o} A_{\text{term}} M_{\text{subst}}. P A \Rightarrow \text{pushprop\_p\_and\_p\_prime } A M P Q \Rightarrow Q \text{ one}$$

Same formula in array format:

$$\begin{aligned} & \forall P_{\text{term} \rightarrow o} Q_{\text{term} \rightarrow o} A_{\text{term}} M_{\text{subst}}. P A \Rightarrow \text{pushprop\_p\_and\_p\_prime} \\ & A M P Q \Rightarrow Q \text{ one} \end{aligned}$$

pushprop\_lem1v2.gthm is an abbreviation defined by

$$\text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \Rightarrow \text{axmap} \Rightarrow \text{axidr}$$

Same formula in array format:

$$\begin{aligned} & \text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \\ & \Rightarrow \text{axmap} \Rightarrow \text{axidr} \Rightarrow \text{axvarshift} \Rightarrow \text{axscons} \Rightarrow \text{ulamvar1} \Rightarrow \text{ulamvarsh} \Rightarrow \text{ulamvarind} \\ & \Rightarrow \text{apinj1} \Rightarrow \text{apinj2} \Rightarrow \text{laminj} \Rightarrow \text{shinj} \Rightarrow \text{lamnotap} \Rightarrow \text{apnotvar} \Rightarrow \text{lamnotvar} \Rightarrow \text{induction} \\ & \Rightarrow \text{pushprop\_lem1v2} \end{aligned}$$

pushprop\_lem1v2.lthm is an abbreviation defined by

$$\text{axvarcons} \Rightarrow \text{pushprop\_lem1v2}$$

pushprop\_lem2v2 is an abbreviation defined by

$$\forall P_{\text{term} \rightarrow o} Q_{\text{term} \rightarrow o} A_{\text{term}} M_{\text{subst}}. \text{pushprop\_p\_and\_p\_prime } A M P Q \Rightarrow (\forall B_{\text{term}}. \text{var } B \Rightarrow P (\text{sub } B M)) \Rightarrow \forall C_{\text{term}}.$$

Same formula in array format:

$$\begin{aligned} & \forall P_{\text{term} \rightarrow o} Q_{\text{term} \rightarrow o} A_{\text{term}} M_{\text{subst}}. \text{pushprop\_p\_and\_p\_prime} \\ & A M P Q \Rightarrow (\forall B_{\text{term}}. \text{var } B \Rightarrow P (\text{sub } B M)) \Rightarrow \forall C_{\text{term}}. \text{var } C \Rightarrow Q C \Rightarrow Q (\text{sub} \\ & C \text{ sh}) \end{aligned}$$

`pushprop_lem2v2_gthm` is an abbreviation defined by

`axapp  $\Rightarrow$  axvarcons  $\Rightarrow$  axvarid  $\Rightarrow$  axabs  $\Rightarrow$  axclos  $\Rightarrow$  axidl  $\Rightarrow$  axshiftcons  $\Rightarrow$  axassoc  $\Rightarrow$  axmap  $\Rightarrow$  axidr`

*Same formula in array format:*

`axapp  $\Rightarrow$  axvarcons  $\Rightarrow$  axvarid  $\Rightarrow$  axabs  $\Rightarrow$  axclos  $\Rightarrow$  axidl  $\Rightarrow$  axshiftcons  $\Rightarrow$  axassoc  
 $\Rightarrow$  axmap  $\Rightarrow$  axidr  $\Rightarrow$  axvarshift  $\Rightarrow$  axscons  $\Rightarrow$  ulamvar1  $\Rightarrow$  ulamvarsh  $\Rightarrow$  ulamvarind  
 $\Rightarrow$  apinj1  $\Rightarrow$  apinj2  $\Rightarrow$  laminj  $\Rightarrow$  shinj  $\Rightarrow$  lamnotap  $\Rightarrow$  apnotvar  $\Rightarrow$  lamnotvar  $\Rightarrow$  induction  
 $\Rightarrow$  pushprop_lem2v2`

`pushprop_lem2v2_lthm` is an abbreviation defined by

`axclos  $\Rightarrow$  axshiftcons  $\Rightarrow$  pushprop_lem2v2`

`pushprop_lem3v2` is an abbreviation defined by

$\forall P_{\text{term} \rightarrow o} Q_{\text{term} \rightarrow o} A_{\text{term}} M_{\text{subst}}. \text{pushprop\_p\_and\_p\_prime } A M P Q \Rightarrow (\forall B_{\text{term}}. \text{var } B \Rightarrow Q B) \Rightarrow \forall B_{\text{term}}. \text{var } B \Rightarrow$

*Same formula in array format:*

$\forall P_{\text{term} \rightarrow o} Q_{\text{term} \rightarrow o} A_{\text{term}} M_{\text{subst}}. \text{pushprop\_p\_and\_p\_prime}$   
 $A M P Q \Rightarrow (\forall B_{\text{term}}. \text{var } B \Rightarrow Q B) \Rightarrow \forall B_{\text{term}}. \text{var } B \Rightarrow P (\text{sub } B (\text{push } A M))$

`pushprop_lem3v2_lthm` is an abbreviation defined by

`pushprop_lem3v2`

`pushprop` is an abbreviation defined by

$\forall P_{\text{term} \rightarrow o} A_{\text{term}} M_{\text{subst}}. (\forall B_{\text{term}}. \text{var } B \Rightarrow P (\text{sub } B M)) \Rightarrow P A \Rightarrow \forall B_{\text{term}}. \text{var } B \Rightarrow P (\text{sub } B (\text{push } A M))$

*Same formula in array format:*

$\forall P_{\text{term} \rightarrow o} A_{\text{term}} M_{\text{subst}}. (\forall B_{\text{term}}. \text{var } B \Rightarrow P (\text{sub } B M))$   
 $\Rightarrow P A \Rightarrow \forall B_{\text{term}}. \text{var } B \Rightarrow P (\text{sub } B (\text{push } A M))$

`pushprop_gthm` is an abbreviation defined by

`axapp  $\Rightarrow$  axvarcons  $\Rightarrow$  axvarid  $\Rightarrow$  axabs  $\Rightarrow$  axclos  $\Rightarrow$  axidl  $\Rightarrow$  axshiftcons  $\Rightarrow$  axassoc  $\Rightarrow$  axmap  $\Rightarrow$  axidr`

*Same formula in array format:*

`axapp  $\Rightarrow$  axvarcons  $\Rightarrow$  axvarid  $\Rightarrow$  axabs  $\Rightarrow$  axclos  $\Rightarrow$  axidl  $\Rightarrow$  axshiftcons  $\Rightarrow$  axassoc  
 $\Rightarrow$  axmap  $\Rightarrow$  axidr  $\Rightarrow$  axvarshift  $\Rightarrow$  axscons  $\Rightarrow$  ulamvar1  $\Rightarrow$  ulamvarsh  $\Rightarrow$  ulamvarind  
 $\Rightarrow$  apinj1  $\Rightarrow$  apinj2  $\Rightarrow$  laminj  $\Rightarrow$  shinj  $\Rightarrow$  lamnotap  $\Rightarrow$  apnotvar  $\Rightarrow$  lamnotvar  $\Rightarrow$  induction  
 $\Rightarrow$  pushprop`

`pushprop_lthm` is an abbreviation defined by

`pushprop_lem0 ⇒ ulamvar1 ⇒ axvarcons ⇒ axclos ⇒ axshiftcons ⇒ ulamvarind ⇒ pushprop`

*Same formula in array format:*

`pushprop_lem0 ⇒ ulamvar1 ⇒ axvarcons ⇒ axclos ⇒ axshiftcons ⇒ ulamvarind  
⇒ pushprop`

`induction2lem` is an abbreviation defined by

$\forall P_{\text{term} \rightarrow o}. (\forall A_{\text{term}} B_{\text{term}}. P A \Rightarrow P B \Rightarrow P(\text{ap } A B)) \Rightarrow (\forall A_{\text{term}}. (\forall B_{\text{term}}. P B \Rightarrow P(\text{sub } A (\text{push } B \text{ id}))) \Rightarrow P(\text{lam } A)) \Rightarrow P(\text{lam } A)$

*Same formula in array format:*

$\forall P_{\text{term} \rightarrow o}. (\forall A_{\text{term}} B_{\text{term}}. P A \Rightarrow P B \Rightarrow P(\text{ap } A B)) \Rightarrow (\forall A_{\text{term}}  
.( \forall B_{\text{term}}. P B \Rightarrow P(\text{sub } A (\text{push } B \text{ id}))) \Rightarrow P(\text{lam } A)) \Rightarrow \forall A_{\text{term}} M_{\text{subst}}  
.( \forall B_{\text{term}}. \text{var } B \Rightarrow P(\text{sub } B M)) \Rightarrow P(\text{sub } A M)$

`induction2lem_gthm` is an abbreviation defined by

`axapp ⇒ axvarcons ⇒ axvarid ⇒ axabs ⇒ axclos ⇒ axidl ⇒ axshiftcons ⇒ axassoc ⇒ axmap ⇒ axidr`

*Same formula in array format:*

`axapp ⇒ axvarcons ⇒ axvarid ⇒ axabs ⇒ axclos ⇒ axidl ⇒ axshiftcons ⇒ axassoc  
⇒ axmap ⇒ axidr ⇒ axvarshift ⇒ axscons ⇒ ulamvar1 ⇒ ulamvarsh ⇒ ulamvarind  
⇒ apinj1 ⇒ apinj2 ⇒ laminj ⇒ shinj ⇒ lamnotap ⇒ apnotvar ⇒ lamnotvar ⇒ induction  
⇒ pushprop ⇒ induction2lem`

`induction2lem_lthm` is an abbreviation defined by

`axapp ⇒ axvarcons ⇒ axabs ⇒ axclos ⇒ axshiftcons ⇒ axassoc ⇒ axmap ⇒ axidr ⇒ induction ⇒ pushprop`

*Same formula in array format:*

`axapp ⇒ axvarcons ⇒ axabs ⇒ axclos ⇒ axshiftcons ⇒ axassoc ⇒ axmap ⇒ axidr ⇒  
induction ⇒ pushprop ⇒ induction2lem`

`induction2` is an abbreviation defined by

$\forall P_{\text{term} \rightarrow o}. (\forall A_{\text{term}}. \text{var } A \Rightarrow P A) \Rightarrow (\forall A_{\text{term}} B_{\text{term}}. P A \Rightarrow P B \Rightarrow P(\text{ap } A B)) \Rightarrow (\forall A_{\text{term}}. (\forall B_{\text{term}}. P B \Rightarrow P(\text{sub } A (\text{push } B \text{ id}))) \Rightarrow P(\text{lam } A)) \Rightarrow \forall A_{\text{term}}. P A$

*Same formula in array format:*

$\forall P_{\text{term} \rightarrow o}. (\forall A_{\text{term}}. \text{var } A \Rightarrow P A) \Rightarrow (\forall A_{\text{term}} B_{\text{term}}. P A \Rightarrow  
P B \Rightarrow P(\text{ap } A B)) \Rightarrow (\forall A_{\text{term}}. (\forall B_{\text{term}}. P B \Rightarrow P(\text{sub } A (\text{push } B \text{ id}))) \Rightarrow P  
(\text{lam } A)) \Rightarrow \forall A_{\text{term}}. P A$

`induction2_gthm` is an abbreviation defined by

`axapp ⇒ axvarcons ⇒ axvarid ⇒ axabs ⇒ axclos ⇒ axidl ⇒ axshiftcons ⇒ axassoc ⇒ axmap ⇒ axidr`

*Same formula in array format:*

`axapp ⇒ axvarcons ⇒ axvarid ⇒ axabs ⇒ axclos ⇒ axidl ⇒ axshiftcons ⇒ axassoc  
⇒ axmap ⇒ axidr ⇒ axvarshift ⇒ axscons ⇒ ulamvar1 ⇒ ulamvarsh ⇒ ulamvarind  
⇒ apinj1 ⇒ apinj2 ⇒ laminj ⇒ shinj ⇒ lamnotap ⇒ apnotvar ⇒ lamnotvar ⇒ induction  
⇒ pushprop ⇒ induction2lem ⇒ induction2`

`induction2_lthm` is an abbreviation defined by

`axvarid ⇒ induction2lem ⇒ induction2`

`substmonoid` is an abbreviation defined by

$(\forall M_{\text{subst}} N_{\text{subst}} K_{\text{subst}}. \text{comp}(\text{comp } M \ N) \ K = \text{comp } M (\text{comp } N \ K)) \wedge (\forall M_{\text{subst}}. \text{comp id } M = M) \wedge \forall M_{\text{subst}}. \text{comp } M \text{ id} = M$

*Same formula in array format:*

$(\forall M_{\text{subst}} N_{\text{subst}} K_{\text{subst}}. \text{comp}(\text{comp } M \ N) \ K = \text{comp } M (\text{comp } N \ K)) \wedge (\forall M_{\text{subst}}. \text{comp id } M = M) \wedge \forall M_{\text{subst}}. \text{comp } M \text{ id} = M$

`substmonoid_gthm` is an abbreviation defined by

`axapp ⇒ axvarcons ⇒ axvarid ⇒ axabs ⇒ axclos ⇒ axidl ⇒ axshiftcons ⇒ axassoc ⇒ axmap ⇒ axidr`

*Same formula in array format:*

`axapp ⇒ axvarcons ⇒ axvarid ⇒ axabs ⇒ axclos ⇒ axidl ⇒ axshiftcons ⇒ axassoc  
⇒ axmap ⇒ axidr ⇒ axvarshift ⇒ axscons ⇒ ulamvar1 ⇒ ulamvarsh ⇒ ulamvarind  
⇒ apinj1 ⇒ apinj2 ⇒ laminj ⇒ shinj ⇒ lamnotap ⇒ apnotvar ⇒ lamnotvar ⇒ induction  
⇒ pushprop ⇒ induction2lem ⇒ induction2 ⇒ substmonoid`

`substmonoid_lthm` is an abbreviation defined by

`axidl ⇒ axassoc ⇒ axidr ⇒ substmonoid`

`termmset` is an abbreviation defined by

$(\forall A_{\text{term}} M_{\text{subst}} N_{\text{subst}}. \text{sub}(\text{sub } A \ M) \ N = \text{sub } A (\text{comp } M \ N)) \wedge \forall A_{\text{term}}. \text{sub } A \text{ id} = A$

*Same formula in array format:*

$(\forall A_{\text{term}} M_{\text{subst}} N_{\text{subst}}. \text{sub}(\text{sub } A \ M) \ N = \text{sub } A (\text{comp } M \ N)) \wedge \forall A_{\text{term}}. \text{sub } A \text{ id} = A$

`termmset_gthm` is an abbreviation defined by

`axapp ⇒ axvarcons ⇒ axvarid ⇒ axabs ⇒ axclos ⇒ axidl ⇒ axshiftcons ⇒ axassoc ⇒ axmap ⇒ axidr`

*Same formula in array format:*

`axapp ⇒ axvarcons ⇒ axvarid ⇒ axabs ⇒ axclos ⇒ axidl ⇒ axshiftcons ⇒ axassoc  
⇒ axmap ⇒ axidr ⇒ axvarshift ⇒ axscons ⇒ ulamvar1 ⇒ ulamvarsh ⇒ ulamvarind  
⇒ apinj1 ⇒ apinj2 ⇒ laminj ⇒ shinj ⇒ lamnotap ⇒ apnotvar ⇒ lamnotvar ⇒ induction  
⇒ pushprop ⇒ induction2lem ⇒ induction2 ⇒ substmonoid ⇒ termmset`

`termmset_lthm` is an abbreviation defined by

`axvarid ⇒ axclos ⇒ termmset`

`hoasap` is an abbreviation defined by

$\lambda M_{\text{subst}} A_{\text{term}} N_{\text{subst}} B_{\text{term}}. \text{ap} (\text{sub } A \ N) \ B$

`hoaslam` is an abbreviation defined by

$\lambda M_{\text{subst}} F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}}. \text{lam} (F \ \text{sh one})$

`hoasvar` is an abbreviation defined by

$\lambda M_{\text{subst}} A_{\text{term}} N_{\text{subst}}. \text{var} (\text{sub } A \ N)$

`hoasapinj1` is an abbreviation defined by

$\forall A_{\text{term}} B_{\text{term}} C_{\text{term}} D_{\text{term}}. \text{hoasap id } A \ \text{id } C = \text{hoasap id } B \ \text{id } D \Rightarrow A = B$

*Same formula in array format:*

$\forall A_{\text{term}} B_{\text{term}} C_{\text{term}} D_{\text{term}}. \text{hoasap id } A \ \text{id } C = \text{hoasap id } B \ \text{id } D \Rightarrow A = B$

`hoasapinj1_gthm` is an abbreviation defined by

`axapp ⇒ axvarcons ⇒ axvarid ⇒ axabs ⇒ axclos ⇒ axidl ⇒ axshiftcons ⇒ axassoc ⇒ axmap ⇒ axidr`

*Same formula in array format:*

`axapp ⇒ axvarcons ⇒ axvarid ⇒ axabs ⇒ axclos ⇒ axidl ⇒ axshiftcons ⇒ axassoc  
⇒ axmap ⇒ axidr ⇒ axvarshift ⇒ axscons ⇒ ulamvar1 ⇒ ulamvarsh ⇒ ulamvarind  
⇒ apinj1 ⇒ apinj2 ⇒ laminj ⇒ shinj ⇒ lamnotap ⇒ apnotvar ⇒ lamnotvar ⇒ induction  
⇒ pushprop ⇒ induction2lem ⇒ induction2 ⇒ substmonoid ⇒ termmset ⇒ hoasapinj1`



hoasapinj1\_lthm is an abbreviation defined by

$$\text{axvarid} \Rightarrow \text{apinj1} \Rightarrow \text{hoasapinj1}$$

hoasapinj2 is an abbreviation defined by

$$\forall A_{\text{term}} B_{\text{term}} C_{\text{term}} D_{\text{term}}. \text{hoasap id } A \text{ id } C = \text{hoasap id } B \text{ id } D \Rightarrow C = D$$

Same formula in array format:

$$\forall A_{\text{term}} B_{\text{term}} C_{\text{term}} D_{\text{term}}. \text{hoasap id } A \text{ id } C = \text{hoasap id } B \text{ id } D \Rightarrow C = D$$

hoasapinj2\_gthm is an abbreviation defined by

$$\text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \Rightarrow \text{axmap} \Rightarrow \text{axidr}$$

Same formula in array format:

$$\begin{aligned} & \text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \\ & \Rightarrow \text{axmap} \Rightarrow \text{axidr} \Rightarrow \text{axvarshift} \Rightarrow \text{axscons} \Rightarrow \text{ulamvar1} \Rightarrow \text{ulamvarsh} \Rightarrow \text{ulamvarind} \\ & \Rightarrow \text{apinj1} \Rightarrow \text{apinj2} \Rightarrow \text{laminj} \Rightarrow \text{shinj} \Rightarrow \text{lamnotap} \Rightarrow \text{apnotvar} \Rightarrow \text{lamnotvar} \Rightarrow \text{induction} \\ & \Rightarrow \text{pushprop} \Rightarrow \text{induction2lem} \Rightarrow \text{induction2} \Rightarrow \text{substmonoid} \Rightarrow \text{termmset} \Rightarrow \text{hoasapinj1} \\ & \Rightarrow \text{hoasapinj2} \end{aligned}$$

hoasapinj2\_lthm is an abbreviation defined by

$$\text{apinj2} \Rightarrow \text{hoasapinj2}$$

hoaslaminj is an abbreviation defined by

$$\forall F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}}. (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}}. \text{sub } (F M A) N = F (\text{comp } M N) (\text{sub } A N)) \Rightarrow \forall G_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}}. (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}}. \text{sub } (G M A) N = G (\text{comp } M N) (\text{sub } A N)) \Rightarrow \text{hoaslaminj id } (\lambda M_{\text{subst}} A_{\text{term}}. F M A) = \text{hoaslaminj id } (\lambda M_{\text{subst}} A_{\text{term}}. G M A) \Rightarrow \forall M_{\text{subst}} A_{\text{term}}. F M A = G M A$$

Same formula in array format:

$$\begin{aligned} & \forall F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}}. (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}}. \text{sub } (F M A) N = F (\text{comp } M N) (\text{sub } A N)) \Rightarrow \forall G_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}}. \\ & (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}}. \text{sub } (G M A) N = G (\text{comp } M N) (\text{sub } A N)) \Rightarrow \text{hoaslaminj id } (\lambda M_{\text{subst}} A_{\text{term}}. F M A) = \text{hoaslaminj id } (\lambda M_{\text{subst}} A_{\text{term}}. G M A) \\ & \Rightarrow \forall M_{\text{subst}} A_{\text{term}}. F M A = G M A \end{aligned}$$

hoaslaminj\_gthm is an abbreviation defined by

$$\text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \Rightarrow \text{axmap} \Rightarrow \text{axidr}$$

Same formula in array format:

$$\begin{aligned}
& \text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \\
& \Rightarrow \text{axmap} \Rightarrow \text{axidr} \Rightarrow \text{axvarshift} \Rightarrow \text{axscons} \Rightarrow \text{ulamvar1} \Rightarrow \text{ulamvarsh} \Rightarrow \text{ulamvarind} \\
& \Rightarrow \text{apinj1} \Rightarrow \text{apinj2} \Rightarrow \text{laminj} \Rightarrow \text{shinj} \Rightarrow \text{lamnotap} \Rightarrow \text{apnotvar} \Rightarrow \text{lamnotvar} \Rightarrow \text{induction} \\
& \Rightarrow \text{pushprop} \Rightarrow \text{induction2lem} \Rightarrow \text{induction2} \Rightarrow \text{substmonoid} \Rightarrow \text{termmset} \Rightarrow \text{hoasapinj1} \\
& \Rightarrow \text{hoasapinj2} \Rightarrow \text{hoaslaminj}
\end{aligned}$$

hoaslaminj\_lthm is an abbreviation defined by

$$\text{axvarcons} \Rightarrow \text{axshiftcons} \Rightarrow \text{laminj} \Rightarrow \text{hoaslaminj}$$

hoaslamnotap is an abbreviation defined by

$$\forall F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}}. (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}}. \text{sub}(F M A) N = F(\text{comp } M N)(\text{sub } A N)) \Rightarrow \forall A_{\text{term}} B_{\text{term}}. \text{hoaslamnotap } A B$$

Same formula in array format:

$$\begin{aligned}
& \forall F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}}. (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}} \\
& . \text{sub}(F M A) N = F(\text{comp } M N)(\text{sub } A N)) \Rightarrow \forall A_{\text{term}} B_{\text{term}}. \text{hoaslamnotap } A B \\
& \text{id}(\lambda M_{\text{subst}} C_{\text{term}}. F M C) \neq \text{hoasapinj1 } A \text{id } B
\end{aligned}$$

hoaslamnotap\_gthm is an abbreviation defined by

$$\text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \Rightarrow \text{axmap} \Rightarrow \text{axidr} \Rightarrow \text{axvarshift} \Rightarrow \text{axscons} \Rightarrow \text{ulamvar1} \Rightarrow \text{ulamvarsh} \Rightarrow \text{ulamvarind} \Rightarrow \text{apinj1} \Rightarrow \text{apinj2} \Rightarrow \text{laminj} \Rightarrow \text{shinj} \Rightarrow \text{lamnotap} \Rightarrow \text{apnotvar} \Rightarrow \text{lamnotvar} \Rightarrow \text{induction} \Rightarrow \text{pushprop} \Rightarrow \text{induction2lem} \Rightarrow \text{induction2} \Rightarrow \text{substmonoid} \Rightarrow \text{termmset} \Rightarrow \text{hoasapinj1} \Rightarrow \text{hoasapinj2} \Rightarrow \text{hoaslaminj} \Rightarrow \text{hoaslamnotap}$$

Same formula in array format:

$$\begin{aligned}
& \text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \\
& \Rightarrow \text{axmap} \Rightarrow \text{axidr} \Rightarrow \text{axvarshift} \Rightarrow \text{axscons} \Rightarrow \text{ulamvar1} \Rightarrow \text{ulamvarsh} \Rightarrow \text{ulamvarind} \\
& \Rightarrow \text{apinj1} \Rightarrow \text{apinj2} \Rightarrow \text{laminj} \Rightarrow \text{shinj} \Rightarrow \text{lamnotap} \Rightarrow \text{apnotvar} \Rightarrow \text{lamnotvar} \Rightarrow \text{induction} \\
& \Rightarrow \text{pushprop} \Rightarrow \text{induction2lem} \Rightarrow \text{induction2} \Rightarrow \text{substmonoid} \Rightarrow \text{termmset} \Rightarrow \text{hoasapinj1} \\
& \Rightarrow \text{hoasapinj2} \Rightarrow \text{hoaslaminj} \Rightarrow \text{hoaslamnotap}
\end{aligned}$$

hoaslamnotap\_lthm is an abbreviation defined by

$$\text{lamnotap} \Rightarrow \text{hoaslamnotap}$$

hoaslamnotvar is an abbreviation defined by

$$\forall F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}}. (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}}. \text{sub}(F M A) N = F(\text{comp } M N)(\text{sub } A N)) \Rightarrow \neg \text{hoasvarid}(\text{hoaslamnotap } A \text{id } B)$$

Same formula in array format:

$$\begin{aligned}
& \forall F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}}. (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}} \\
& . \text{sub}(F M A) N = F(\text{comp } M N)(\text{sub } A N)) \Rightarrow \neg \text{hoasvarid}(\text{hoaslamnotap } A \text{id } B) \\
& A_{\text{term}}. F M A) \text{id}
\end{aligned}$$

`hoaslamnotvar_gthm` is an abbreviation defined by

`axapp ⇒ axvarcons ⇒ axvarid ⇒ axabs ⇒ axclos ⇒ axidl ⇒ axshiftcons ⇒ axassoc ⇒ axmap ⇒ axidr`

*Same formula in array format:*

`axapp ⇒ axvarcons ⇒ axvarid ⇒ axabs ⇒ axclos ⇒ axidl ⇒ axshiftcons ⇒ axassoc  
 ⇒ axmap ⇒ axidr ⇒ axvarshift ⇒ axscons ⇒ ulamvar1 ⇒ ulamvarsh ⇒ ulamvarind  
 ⇒ apinj1 ⇒ apinj2 ⇒ laminj ⇒ shinj ⇒ lamnotap ⇒ apnotvar ⇒ lamnotvar ⇒ induction  
 ⇒ pushprop ⇒ induction2lem ⇒ induction2 ⇒ substmonoid ⇒ termset ⇒ hoasapinj1  
 ⇒ hoasapinj2 ⇒ hoaslaminj ⇒ hoaslamnotap ⇒ hoaslamnotvar`

`hoaslamnotvar_lthm` is an abbreviation defined by

`axvarid ⇒ lamnotvar ⇒ hoaslamnotvar`

`hoasapnotvar` is an abbreviation defined by

$\forall A_{\text{term}} B_{\text{term}}. \neg \text{hoasvar id} (\text{hoasap id } A \text{ id } B) \text{ id}$

`hoasapnotvar_gthm` is an abbreviation defined by

`axapp ⇒ axvarcons ⇒ axvarid ⇒ axabs ⇒ axclos ⇒ axidl ⇒ axshiftcons ⇒ axassoc ⇒ axmap ⇒ axidr`

*Same formula in array format:*

`axapp ⇒ axvarcons ⇒ axvarid ⇒ axabs ⇒ axclos ⇒ axidl ⇒ axshiftcons ⇒ axassoc  
 ⇒ axmap ⇒ axidr ⇒ axvarshift ⇒ axscons ⇒ ulamvar1 ⇒ ulamvarsh ⇒ ulamvarind  
 ⇒ apinj1 ⇒ apinj2 ⇒ laminj ⇒ shinj ⇒ lamnotap ⇒ apnotvar ⇒ lamnotvar ⇒ induction  
 ⇒ pushprop ⇒ induction2lem ⇒ induction2 ⇒ substmonoid ⇒ termset ⇒ hoasapinj1  
 ⇒ hoasapinj2 ⇒ hoaslaminj ⇒ hoaslamnotap ⇒ hoaslamnotvar ⇒ hoasapnotvar`

`hoasapnotvar_lthm` is an abbreviation defined by

`axvarid ⇒ apnotvar ⇒ hoasapnotvar`

`hoasinduction_p_and_p_prime` is an abbreviation defined by

$\lambda P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o} Q_{\text{term} \rightarrow o}. \forall X_{\text{term}}. Q X \Leftrightarrow P \text{ id } X \text{ id}$

*Same formula in array format:*

$\lambda P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o} Q_{\text{term} \rightarrow o}. \forall X_{\text{term}} \\ . Q X \Leftrightarrow P \text{ id } X \text{ id}$

`hoasinduction_lem0` is an abbreviation defined by

$$\forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o}. \exists Q_{\text{term} \rightarrow o}. \text{hoasinduction\_p\_and\_p\_prime } P Q$$

*Same formula in array format:*

$$\forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o}. \exists Q_{\text{term} \rightarrow o}. \text{hoasinduction\_p\_and\_p\_prime } P Q$$

`hoasinduction_lem0_lthm` is an abbreviation defined by

$$\text{hoasinduction\_lem0}$$

`hoasinduction_lem1v2` is an abbreviation defined by

$$\forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o} Q_{\text{term} \rightarrow o}. (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}} K_{\text{subst}}. P M A (\text{comp } K N) \Rightarrow P (\text{comp } M K) (\text{sub } A K) N) \Rightarrow$$

*Same formula in array format:*

$$\begin{aligned} & \forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o} Q_{\text{term} \rightarrow o}. (\forall M_{\text{subst}} \\ & A_{\text{term}} N_{\text{subst}} K_{\text{subst}}. P M A (\text{comp } K N) \Rightarrow P (\text{comp } M K) (\text{sub } A \\ & K) N) \Rightarrow (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}} K_{\text{subst}}. P (\text{comp } M K) (\text{sub } \\ & A K) N \Rightarrow P M A (\text{comp } K N)) \Rightarrow (\forall A_{\text{term}}. \text{hoasvarid } A \text{id} \Rightarrow P \text{id } A \text{id}) \Rightarrow \text{hoasinduction\_p\_and\_p\_prime} \\ & P Q \Rightarrow \forall A_{\text{term}}. \text{var } A \Rightarrow Q A \end{aligned}$$

`hoasinduction_lem1v2_gthm` is an abbreviation defined by

$$\text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \Rightarrow \text{axmap} \Rightarrow \text{axidr}$$

*Same formula in array format:*

$$\begin{aligned} & \text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \\ & \Rightarrow \text{axmap} \Rightarrow \text{axidr} \Rightarrow \text{axvarshift} \Rightarrow \text{axscons} \Rightarrow \text{ulamvar1} \Rightarrow \text{ulamvarsh} \Rightarrow \text{ulamvarind} \\ & \Rightarrow \text{apinj1} \Rightarrow \text{apinj2} \Rightarrow \text{laminj} \Rightarrow \text{shinj} \Rightarrow \text{lamnotap} \Rightarrow \text{apnotvar} \Rightarrow \text{lamnotvar} \Rightarrow \text{induction} \\ & \Rightarrow \text{pushprop} \Rightarrow \text{induction2lem} \Rightarrow \text{induction2} \Rightarrow \text{substmonoid} \Rightarrow \text{termmset} \Rightarrow \text{hoasapinj1} \\ & \Rightarrow \text{hoasapinj2} \Rightarrow \text{hoaslaminj} \Rightarrow \text{hoaslamnotap} \Rightarrow \text{hoaslamnotvar} \Rightarrow \text{hoasapnotvar} \\ & \Rightarrow \text{hoasinduction\_lem1v2} \end{aligned}$$

`hoasinduction_lem2v2` is an abbreviation defined by

$$\forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o} Q_{\text{term} \rightarrow o}. (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}} K_{\text{subst}}. P M A (\text{comp } K N) \Rightarrow P (\text{comp } M K) (\text{sub } A K) N) \Rightarrow$$

Same formula in array format:

$$\begin{aligned}
& \forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o} Q_{\text{term} \rightarrow o} . (\forall M_{\text{subst}} \\
& A_{\text{term}} N_{\text{subst}} K_{\text{subst}} . P \ M \ A \ (\text{comp } K \ N) \Rightarrow P \ (\text{comp } M \ K) \ (\text{sub } A \\
& K) \ N) \Rightarrow (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}} K_{\text{subst}} . P \ (\text{comp } M \ K) \ (\text{sub} \\
& A \ K) \ N \Rightarrow P \ M \ A \ (\text{comp } K \ N)) \Rightarrow (\forall A_{\text{term}} B_{\text{term}} . P \ \text{id } A \ \text{id} \Rightarrow P \ \text{id } B \ \text{id} \Rightarrow P \\
& \text{id } (\text{hoasap id } A \ \text{id } B) \ \text{id}) \Rightarrow \text{hoasinduction\_p\_and\_p\_prime } P \ Q \Rightarrow \forall A_{\text{term}} \\
& B_{\text{term}} . Q \ A \Rightarrow Q \ B \Rightarrow Q \ (\text{ap } A \ B)
\end{aligned}$$

hoasinduction\_lem2v2\_gthm is an abbreviation defined by

axapp  $\Rightarrow$  axvarcons  $\Rightarrow$  axvarid  $\Rightarrow$  axabs  $\Rightarrow$  axclos  $\Rightarrow$  axidl  $\Rightarrow$  axshiftcons  $\Rightarrow$  axassoc  $\Rightarrow$  axmap  $\Rightarrow$  axidr

Same formula in array format:

$$\begin{aligned}
& \text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \\
& \Rightarrow \text{axmap} \Rightarrow \text{axidr} \Rightarrow \text{axvarshift} \Rightarrow \text{axscons} \Rightarrow \text{ulamvar1} \Rightarrow \text{ulamvarsh} \Rightarrow \text{ulamvarind} \\
& \Rightarrow \text{apinj1} \Rightarrow \text{apinj2} \Rightarrow \text{laminj} \Rightarrow \text{shinj} \Rightarrow \text{lamnotap} \Rightarrow \text{apnotvar} \Rightarrow \text{lamnotvar} \Rightarrow \text{induction} \\
& \Rightarrow \text{pushprop} \Rightarrow \text{induction2lem} \Rightarrow \text{induction2} \Rightarrow \text{substmonoid} \Rightarrow \text{termmset} \Rightarrow \text{hoasapinj1} \\
& \Rightarrow \text{hoasapinj2} \Rightarrow \text{hoaslaminj} \Rightarrow \text{hoaslamnotap} \Rightarrow \text{hoaslamnotvar} \Rightarrow \text{hoasapnotvar} \\
& \Rightarrow \text{hoasinduction\_lem2v2}
\end{aligned}$$

hoasinduction\_lem3v2\_f is an abbreviation defined by

$$\forall B_{\text{term}} . \exists F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}} . \forall A_{\text{term}} M_{\text{subst}} . F \ M \ A = \text{sub } B \ (\text{push } A \ M)$$

Same formula in array format:

$$\begin{aligned}
& \forall B_{\text{term}} . \exists F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}} . \forall A_{\text{term}} M_{\text{subst}} \\
& . F \ M \ A = \text{sub } B \ (\text{push } A \ M)
\end{aligned}$$

hoasinduction\_lem3v2\_f\_lthm is an abbreviation defined by

$$\text{hoasinduction\_lem3v2\_f}$$

hoasinduction\_lem3v2 is an abbreviation defined by

$$\forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o} Q_{\text{term} \rightarrow o} . (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}} K_{\text{subst}} . P \ M \ A \ (\text{comp } K \ N) \Rightarrow P \ (\text{comp } M \ K) \ (\text{sub } A \ K) \ N) =$$

Same formula in array format:

$$\begin{aligned}
& \forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o} Q_{\text{term} \rightarrow o} \cdot (\forall M_{\text{subst}} \\
& A_{\text{term}} N_{\text{subst}} K_{\text{subst}} \cdot P M A (\text{comp } K N) \Rightarrow P (\text{comp } M K) (\text{sub } A \\
& K) N) \Rightarrow (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}} K_{\text{subst}} \cdot P (\text{comp } M K) (\text{sub} \\
& A K) N \Rightarrow P M A (\text{comp } K N)) \Rightarrow (\forall F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}} \cdot (\forall M_{\text{subst}} \\
& A_{\text{term}} N_{\text{subst}} \cdot \text{sub } (F M A) N = F (\text{comp } M N) (\text{sub } A N)) \Rightarrow (\forall A_{\text{term}} \\
& \cdot P \text{id } A \text{id} \Rightarrow P \text{id } (F \text{id } A) \text{id} \Rightarrow P \text{id } (\text{hoaslamid } \lambda M_{\text{subst}} A_{\text{term}} \cdot \\
& F M A) \text{id}) \Rightarrow \text{hoasinduction\_p\_and\_p\_prime } P Q \Rightarrow \forall A_{\text{term}} \cdot (\forall B_{\text{term}} \\
& \cdot Q B \Rightarrow Q (\text{sub } A (\text{push } B \text{id}))) \Rightarrow Q (\text{lam } A)
\end{aligned}$$

hoasinduction\_lem3v2\_gthm is an abbreviation defined by

axapp  $\Rightarrow$  axvarcons  $\Rightarrow$  axvarid  $\Rightarrow$  axabs  $\Rightarrow$  axclos  $\Rightarrow$  axidl  $\Rightarrow$  axshiftcons  $\Rightarrow$  axassoc  $\Rightarrow$  axmap  $\Rightarrow$  axidr

Same formula in array format:

$$\begin{aligned}
& \text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \\
& \Rightarrow \text{axmap} \Rightarrow \text{axidr} \Rightarrow \text{axvarshift} \Rightarrow \text{axscons} \Rightarrow \text{ulamvar1} \Rightarrow \text{ulamvarsh} \Rightarrow \text{ulamvarind} \\
& \Rightarrow \text{apinj1} \Rightarrow \text{apinj2} \Rightarrow \text{laminj} \Rightarrow \text{shinj} \Rightarrow \text{lamnotap} \Rightarrow \text{apnotvar} \Rightarrow \text{lamnotvar} \Rightarrow \text{induction} \\
& \Rightarrow \text{pushprop} \Rightarrow \text{induction2lem} \Rightarrow \text{induction2} \Rightarrow \text{substmonoid} \Rightarrow \text{termmset} \Rightarrow \text{hoasapinj1} \\
& \Rightarrow \text{hoasapinj2} \Rightarrow \text{hoaslaminj} \Rightarrow \text{hoaslamnotap} \Rightarrow \text{hoaslamnotvar} \Rightarrow \text{hoasapnotvar} \\
& \Rightarrow \text{hoasinduction\_lem3v2}
\end{aligned}$$

hoasinduction\_lem3v2\_lthm is an abbreviation defined by

$$\text{axvarid} \Rightarrow \text{axvarshift} \Rightarrow \text{axclos} \Rightarrow \text{axmap} \Rightarrow \text{hoasinduction\_lem3v2}$$

hoasinduction\_lem3v2a is an abbreviation defined by

$$\forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o} Q_{\text{term} \rightarrow o} \cdot (\forall F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}} \cdot (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}} \cdot \text{sub } (F M A) N = F (\text{comp } M N) (\text{sub } A$$

Same formula in array format:

$$\begin{aligned}
& \forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o} Q_{\text{term} \rightarrow o} \cdot (\forall F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}} \\
& \cdot (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}} \cdot \text{sub } (F M A) N = F (\text{comp } M N) (\text{sub } A \\
& N)) \Rightarrow (\forall A_{\text{term}} \cdot P \text{id } A \text{id} \Rightarrow P \text{id } (F \text{id } A) \text{id} \Rightarrow P \text{id } (\text{hoaslamid } \lambda M_{\text{subst}} \\
& A_{\text{term}} \cdot F M A) \text{id}) \Rightarrow \text{hoasinduction\_p\_and\_p\_prime } P Q \Rightarrow \forall A_{\text{term}} \\
& \cdot (\forall B_{\text{term}} \cdot Q B \Rightarrow Q (\text{sub } A (\text{push } B \text{id}))) \Rightarrow Q (\text{lam } A)
\end{aligned}$$

`hoasinduction_lem3v2a_lthm` is an abbreviation defined by

`hoasinduction_lem3v2_f`  $\Rightarrow$  `axvarid`  $\Rightarrow$  `axvarshift`  $\Rightarrow$  `axclos`  $\Rightarrow$  `axmap`  $\Rightarrow$  `hoasinduction_lem3v2a`

`hoasinduction_lem1` is an abbreviation defined by

$\forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o}. (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}} K_{\text{subst}}. P M A (\text{comp } K N) \Rightarrow P (\text{comp } M K) (\text{sub } A K) N) \Rightarrow (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}} K_{\text{subst}}. P (\text{comp } M K) (\text{sub } A K) N \Rightarrow P M A (\text{comp } K N)) \Rightarrow (\forall A_{\text{term}} \text{.hoasvarid } A \text{id} \Rightarrow P \text{id } A \text{id}) \Rightarrow \forall A_{\text{term}}. \text{var } A \Rightarrow P \text{id } A \text{id}$

*Same formula in array format:*

$$\begin{aligned} & \forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o}. (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}} \\ & K_{\text{subst}}. P M A (\text{comp } K N) \Rightarrow P (\text{comp } M K) (\text{sub } A K) N) \Rightarrow (\forall M_{\text{subst}} A_{\text{term}} \\ & N_{\text{subst}} K_{\text{subst}}. P (\text{comp } M K) (\text{sub } A K) N \Rightarrow P M A (\text{comp } K N)) \Rightarrow (\forall A_{\text{term}} \\ & \text{.hoasvarid } A \text{id} \Rightarrow P \text{id } A \text{id}) \Rightarrow \forall A_{\text{term}}. \text{var } A \Rightarrow P \text{id } A \text{id} \end{aligned}$$

`hoasinduction_lem1_gthm` is an abbreviation defined by

`axapp`  $\Rightarrow$  `axvarcons`  $\Rightarrow$  `axvarid`  $\Rightarrow$  `axabs`  $\Rightarrow$  `axclos`  $\Rightarrow$  `axidl`  $\Rightarrow$  `axshiftcons`  $\Rightarrow$  `axassoc`  $\Rightarrow$  `axmap`  $\Rightarrow$  `axidr`

*Same formula in array format:*

$$\begin{aligned} & \text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \\ & \Rightarrow \text{axmap} \Rightarrow \text{axidr} \Rightarrow \text{axvarshift} \Rightarrow \text{axscons} \Rightarrow \text{ulamvar1} \Rightarrow \text{ulamvarsh} \Rightarrow \text{ulamvarind} \\ & \Rightarrow \text{apinj1} \Rightarrow \text{apinj2} \Rightarrow \text{laminj} \Rightarrow \text{shinj} \Rightarrow \text{lamnotap} \Rightarrow \text{apnotvar} \Rightarrow \text{lamnotvar} \Rightarrow \text{induction} \\ & \Rightarrow \text{pushprop} \Rightarrow \text{induction2lem} \Rightarrow \text{induction2} \Rightarrow \text{substmonoid} \Rightarrow \text{termmset} \Rightarrow \text{hoasapinj1} \\ & \Rightarrow \text{hoasapinj2} \Rightarrow \text{hoaslaminj} \Rightarrow \text{hoaslamnotap} \Rightarrow \text{hoaslamnotvar} \Rightarrow \text{hoasapnotvar} \\ & \Rightarrow \text{hoasinduction_lem1} \end{aligned}$$

`hoasinduction_lem1_lthm` is an abbreviation defined by

`axapp`  $\Rightarrow$  `axvarcons`  $\Rightarrow$  `axvarid`  $\Rightarrow$  `axabs`  $\Rightarrow$  `axclos`  $\Rightarrow$  `axidl`  $\Rightarrow$  `axshiftcons`  $\Rightarrow$  `axassoc`  $\Rightarrow$  `axmap`  $\Rightarrow$  `axidr`

*Same formula in array format:*

$$\begin{aligned} & \text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \\ & \Rightarrow \text{axmap} \Rightarrow \text{axidr} \Rightarrow \text{axvarshift} \Rightarrow \text{axscons} \Rightarrow \text{ulamvar1} \Rightarrow \text{ulamvarsh} \Rightarrow \text{ulamvarind} \\ & \Rightarrow \text{apinj1} \Rightarrow \text{apinj2} \Rightarrow \text{laminj} \Rightarrow \text{shinj} \Rightarrow \text{lamnotap} \Rightarrow \text{apnotvar} \Rightarrow \text{lamnotvar} \Rightarrow \text{induction} \\ & \Rightarrow \text{pushprop} \Rightarrow \text{induction2lem} \Rightarrow \text{induction2} \Rightarrow \text{substmonoid} \Rightarrow \text{termmset} \Rightarrow \text{hoasapinj1} \\ & \Rightarrow \text{hoasapinj2} \Rightarrow \text{hoaslaminj} \Rightarrow \text{hoaslamnotap} \Rightarrow \text{hoaslamnotvar} \Rightarrow \text{hoasapnotvar} \\ & \Rightarrow \text{hoasinduction_lem1} \end{aligned}$$

`hoasinduction_lem2` is an abbreviation defined by

$\forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o}. (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}} K_{\text{subst}}. P M A (\text{comp } K N) \Rightarrow P (\text{comp } M K) (\text{sub } A K) N) \Rightarrow (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}} K_{\text{subst}}. P (\text{comp } M K) (\text{sub } A K) N \Rightarrow P M A (\text{comp } K N)) \Rightarrow (\forall A_{\text{term}} \text{.hoasvarid } A \text{id} \Rightarrow P \text{id } A \text{id}) \Rightarrow \forall A_{\text{term}}. \text{var } A \Rightarrow P \text{id } A \text{id}$

Same formula in array format:

$$\begin{aligned}
& \forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o}. (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}} \\
& K_{\text{subst}}. P M A (\text{comp } K N) \Rightarrow P (\text{comp } M K) (\text{sub } A K) N) \Rightarrow (\forall M_{\text{subst}} A_{\text{term}} \\
& N_{\text{subst}} K_{\text{subst}}. P (\text{comp } M K) (\text{sub } A K) N \Rightarrow P M A (\text{comp } K N)) \Rightarrow (\forall A_{\text{term}} \\
& B_{\text{term}}. P \text{id } A \text{id} \Rightarrow P \text{id } B \text{id} \Rightarrow P \text{id} (\text{hoasap id } A \text{id } B) \text{id}) \Rightarrow \forall A_{\text{term}} \\
& B_{\text{term}}. P \text{id } A \text{id} \Rightarrow P \text{id } B \text{id} \Rightarrow P \text{id} (\text{ap } A B) \text{id}
\end{aligned}$$

hoasinduction\_lem2\_gthm is an abbreviation defined by

axapp  $\Rightarrow$  axvarcons  $\Rightarrow$  axvarid  $\Rightarrow$  axabs  $\Rightarrow$  axclos  $\Rightarrow$  axidl  $\Rightarrow$  axshiftcons  $\Rightarrow$  axassoc  $\Rightarrow$  axmap  $\Rightarrow$  axidr

Same formula in array format:

$$\begin{aligned}
& \text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \\
& \Rightarrow \text{axmap} \Rightarrow \text{axidr} \Rightarrow \text{axvarshift} \Rightarrow \text{axscons} \Rightarrow \text{ulamvar1} \Rightarrow \text{ulamvarsh} \Rightarrow \text{ulamvarind} \\
& \Rightarrow \text{apinj1} \Rightarrow \text{apinj2} \Rightarrow \text{laminj} \Rightarrow \text{shinj} \Rightarrow \text{lamnotap} \Rightarrow \text{apnotvar} \Rightarrow \text{lamnotvar} \Rightarrow \text{induction} \\
& \Rightarrow \text{pushprop} \Rightarrow \text{induction2lem} \Rightarrow \text{induction2} \Rightarrow \text{substmonoid} \Rightarrow \text{termmset} \Rightarrow \text{hoasapinj1} \\
& \Rightarrow \text{hoasapinj2} \Rightarrow \text{hoaslaminj} \Rightarrow \text{hoaslamnotap} \Rightarrow \text{hoaslamnotvar} \Rightarrow \text{hoasapnotvar} \\
& \Rightarrow \text{hoasinduction\_lem2}
\end{aligned}$$

hoasinduction\_lem2\_lthm is an abbreviation defined by

axapp  $\Rightarrow$  axvarcons  $\Rightarrow$  axvarid  $\Rightarrow$  axabs  $\Rightarrow$  axclos  $\Rightarrow$  axidl  $\Rightarrow$  axshiftcons  $\Rightarrow$  axassoc  $\Rightarrow$  axmap  $\Rightarrow$  axidr

Same formula in array format:

$$\begin{aligned}
& \text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \\
& \Rightarrow \text{axmap} \Rightarrow \text{axidr} \Rightarrow \text{axvarshift} \Rightarrow \text{axscons} \Rightarrow \text{ulamvar1} \Rightarrow \text{ulamvarsh} \Rightarrow \text{ulamvarind} \\
& \Rightarrow \text{apinj1} \Rightarrow \text{apinj2} \Rightarrow \text{laminj} \Rightarrow \text{shinj} \Rightarrow \text{lamnotap} \Rightarrow \text{apnotvar} \Rightarrow \text{lamnotvar} \Rightarrow \text{induction} \\
& \Rightarrow \text{pushprop} \Rightarrow \text{induction2lem} \Rightarrow \text{induction2} \Rightarrow \text{substmonoid} \Rightarrow \text{termmset} \Rightarrow \text{hoasapinj1} \\
& \Rightarrow \text{hoasapinj2} \Rightarrow \text{hoaslaminj} \Rightarrow \text{hoaslamnotap} \Rightarrow \text{hoaslamnotvar} \Rightarrow \text{hoasapnotvar} \\
& \Rightarrow \text{hoasinduction\_lem2}
\end{aligned}$$

hoasinduction\_lem3aa is an abbreviation defined by

$\forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o}. (\forall F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}}. (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}}. \text{sub } (F M A) N = F (\text{comp } M N) (\text{sub } A N)) \Rightarrow$

Same formula in array format:

$$\begin{aligned}
& \forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o}. (\forall F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}} \\
& . (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}}. \text{sub } (F M A) N = F (\text{comp } M N) (\text{sub } A \\
& N)) \Rightarrow (\forall A_{\text{term}}. P \text{id } A \text{id} \Rightarrow P \text{id} (F \text{id } A) \text{id}) \Rightarrow P \text{id} (\text{hoaslam id } \lambda M_{\text{subst}} \\
& A_{\text{term}}. F M A) \text{id}) \Rightarrow \forall A_{\text{term}}. (\forall B_{\text{term}}. P \text{id } B \text{id} \Rightarrow P \text{id} (\text{sub } A \\
& (\text{push } B \text{id})) \text{id}) \Rightarrow P \text{id} (\text{lam } (\text{sub } A (\text{push one sh}))) \text{id}
\end{aligned}$$



hoasinduction\_lem3aa\_lthm is an abbreviation defined by

$$\text{axclos} \Rightarrow \text{axmap} \Rightarrow \text{hoasinduction\_lem3aa}$$

hoasinduction\_lem3aaa is an abbreviation defined by

$$(\forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o}. (\forall F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}}. \exists C_{\text{term}}. \forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}}. \text{sub}(F M A) N = \text{sub}(\text{sub } C (\text{push } A M$$

Same formula in array format:

$$\begin{aligned}
& (\forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o}. (\forall F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}} \\
& \quad . \exists C_{\text{term}}. \forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}}. \text{sub} (F M A) N = \text{sub} (\text{sub} \\
& C (\text{push } A M)) N \wedge \text{sub } C (\text{push} (\text{sub } A N) (\text{comp } M N)) = F (\text{comp } M N) (\text{sub } A N)) \\
& \Rightarrow (\forall A_{\text{term}}. P \text{id } A \text{id} \Rightarrow P \text{id} (F \text{id } A) \text{id}) \Rightarrow P \text{id} (\text{hoaslam id } \lambda M_{\text{subst}} \\
& \quad A_{\text{term}}. F M A) \text{id} \Rightarrow \forall A_{\text{term}}. (\forall B_{\text{term}}. P \text{id } B \text{id} \Rightarrow P \text{id} (\text{sub } A \\
& \quad (\text{push } B \text{id})) \text{id}) \Rightarrow P \text{id} (\text{lam} (\text{sub } A (\text{push one sh})) \text{id}
\end{aligned}$$

`hoasinduction_lem3` is an abbreviation defined by

$$\forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o}. (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}} K_{\text{subst}}. P M A (\text{comp } K N) \Rightarrow P (\text{comp } M K) (\text{sub } A K) N) \Rightarrow (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}} K_{\text{subst}}. P M A (\text{comp } K N) \Rightarrow P (\text{comp } M K) (\text{sub } A K) N)$$

Same formula in array format:

$$\begin{aligned} & \forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o}. (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}} \\ & K_{\text{subst}}. P M A (\text{comp } K N) \Rightarrow P (\text{comp } M K) (\text{sub } A K) N) \Rightarrow (\forall M_{\text{subst}} A_{\text{term}} \\ & N_{\text{subst}} K_{\text{subst}}. P (\text{comp } M K) (\text{sub } A K) N \Rightarrow P M A (\text{comp } K N)) \Rightarrow (\forall F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}} \\ & . (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}}. \text{sub } (F M A) N = F (\text{comp } M N) (\text{sub } A \\ & N)) \Rightarrow (\forall A_{\text{term}}. P \text{id } A \text{id} \Rightarrow P \text{id } (F \text{id } A) \text{id}) \Rightarrow P \text{id } (\text{hoaslam id } \lambda M_{\text{subst}} \\ & A_{\text{term}}. F M A) \text{id}) \Rightarrow \forall A_{\text{term}}. (\forall B_{\text{term}}. P \text{id } B \text{id} \Rightarrow P \text{id } (\text{sub } A \\ & (\text{push } B \text{id})) \text{id}) \Rightarrow P \text{id } (\text{lam } A) \text{id} \end{aligned}$$

hoasinduction\_lem3\_gthm is an abbreviation defined by

$$\text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \Rightarrow \text{axmap} \Rightarrow \text{axidr}$$

Same formula in array format:

$$\begin{aligned} & \text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \\ & \Rightarrow \text{axmap} \Rightarrow \text{axidr} \Rightarrow \text{axvarshift} \Rightarrow \text{axscons} \Rightarrow \text{ulamvar1} \Rightarrow \text{ulamvarsh} \Rightarrow \text{ulamvarind} \\ \Rightarrow & \text{apinj1} \Rightarrow \text{apinj2} \Rightarrow \text{laminj} \Rightarrow \text{shinj} \Rightarrow \text{lamnotap} \Rightarrow \text{apnotvar} \Rightarrow \text{lamnotvar} \Rightarrow \text{induction} \\ & \Rightarrow \text{pushprop} \Rightarrow \text{induction2lem} \Rightarrow \text{induction2} \Rightarrow \text{substmonoid} \Rightarrow \text{termmset} \Rightarrow \text{hoasapinj1} \\ & \Rightarrow \text{hoasapinj2} \Rightarrow \text{hoaslaminj} \Rightarrow \text{hoaslamnotap} \Rightarrow \text{hoaslamnotvar} \Rightarrow \text{hoasapnotvar} \\ & \Rightarrow \text{hoasinduction\_lem1} \Rightarrow \text{hoasinduction\_lem2} \Rightarrow \text{hoasinduction\_lem3} \end{aligned}$$

hoasinduction\_lem3\_lthm is an abbreviation defined by

$$\text{axvarid} \Rightarrow \text{axvarshift} \Rightarrow \text{hoasinduction\_lem3aa} \Rightarrow \text{hoasinduction\_lem3}$$

hoasinduction\_lem3a is an abbreviation defined by

$$\forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o}. (\forall F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}}. (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}}. \text{sub } (F M A) N = F (\text{comp } M N) (\text{sub } A N)) \Rightarrow$$

*Same formula in array format:*

$$\begin{aligned} & \forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o}. (\forall F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}} \\ & \quad . (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}}. \text{sub } (F M A) N = F (\text{comp } M N) (\text{sub } A \\ & \quad N)) \Rightarrow (\forall A_{\text{term}}. P \text{ id } A \text{ id} \Rightarrow P \text{ id } (F \text{ id } A) \text{ id}) \Rightarrow P \text{ id } (\text{hoaslam id } \lambda M_{\text{subst}} \\ & \quad A_{\text{term}}. F M A) \text{ id}) \Rightarrow \forall A_{\text{term}}. (\forall B_{\text{term}}. P \text{ id } B \text{ id} \Rightarrow P \text{ id } (\text{sub } A \\ & \quad (\text{push } B \text{ id})) \text{ id}) \Rightarrow P \text{ id } (\text{lam } A) \text{ id} \end{aligned}$$

hoasinduction\_lem3a\_gthm is an abbreviation defined by

$$\text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \Rightarrow \text{axmap} \Rightarrow \text{axidr}$$

*Same formula in array format:*

$$\begin{aligned} & \text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \\ & \Rightarrow \text{axmap} \Rightarrow \text{axidr} \Rightarrow \text{axvarshift} \Rightarrow \text{axscons} \Rightarrow \text{ulamvar1} \Rightarrow \text{ulamvarsh} \Rightarrow \text{ulamvarind} \\ & \Rightarrow \text{apinj1} \Rightarrow \text{apinj2} \Rightarrow \text{laminj} \Rightarrow \text{shinj} \Rightarrow \text{lamnotap} \Rightarrow \text{apnotvar} \Rightarrow \text{lamnotvar} \Rightarrow \text{induction} \\ & \Rightarrow \text{pushprop} \Rightarrow \text{induction2lem} \Rightarrow \text{induction2} \Rightarrow \text{substmonoid} \Rightarrow \text{termmmset} \Rightarrow \text{hoasapinj1} \\ & \Rightarrow \text{hoasapinj2} \Rightarrow \text{hoaslaminj} \Rightarrow \text{hoaslamnotap} \Rightarrow \text{hoaslamnotvar} \Rightarrow \text{hoasapnotvar} \\ & \Rightarrow \text{hoasinduction\_lem1} \Rightarrow \text{hoasinduction\_lem2} \Rightarrow \text{hoasinduction\_lem3a} \end{aligned}$$

hoasinduction\_lem3a\_lthm is an abbreviation defined by

$$\text{axvarid} \Rightarrow \text{axvarshift} \Rightarrow \text{hoasinduction\_lem3aa} \Rightarrow \text{hoasinduction\_lem3a}$$

hoasinduction\_lem3b is an abbreviation defined by

$$\forall B_{\text{term}}. (\exists F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}}. \text{sub } B (\text{push one sh})) = F \text{ sh one}$$

*Same formula in array format:*

$$\begin{aligned} & \forall B_{\text{term}}. (\exists F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}}. \text{sub } B (\text{push one sh})) \\ & \quad = F \text{ sh one} \end{aligned}$$

hoasinduction\_lem3b\_gthm is an abbreviation defined by

$$\text{axapp} \Rightarrow \text{axvarcons} \Rightarrow \text{axvarid} \Rightarrow \text{axabs} \Rightarrow \text{axclos} \Rightarrow \text{axidl} \Rightarrow \text{axshiftcons} \Rightarrow \text{axassoc} \Rightarrow \text{axmap} \Rightarrow \text{axidr}$$

Same formula in array format:

axapp  $\Rightarrow$  axvarcons  $\Rightarrow$  axvarid  $\Rightarrow$  axabs  $\Rightarrow$  axclos  $\Rightarrow$  axidl  $\Rightarrow$  axshiftcons  $\Rightarrow$  axassoc  
 $\Rightarrow$  axmap  $\Rightarrow$  axidr  $\Rightarrow$  axvarshift  $\Rightarrow$  axscons  $\Rightarrow$  ulamvar1  $\Rightarrow$  ulamvarsh  $\Rightarrow$  ulamvarind  
 $\Rightarrow$  apinj1  $\Rightarrow$  apinj2  $\Rightarrow$  laminj  $\Rightarrow$  shinj  $\Rightarrow$  lamnotap  $\Rightarrow$  apnotvar  $\Rightarrow$  lamnotvar  $\Rightarrow$  induction  
 $\Rightarrow$  pushprop  $\Rightarrow$  induction2lem  $\Rightarrow$  induction2  $\Rightarrow$  substmonoid  $\Rightarrow$  termmmset  $\Rightarrow$  hoasapinj1  
 $\Rightarrow$  hoasapinj2  $\Rightarrow$  hoaslaminj  $\Rightarrow$  hoaslamnotap  $\Rightarrow$  hoaslamnotvar  $\Rightarrow$  hoasapnotvar  
 $\Rightarrow$  hoasinduction\_lem1  $\Rightarrow$  hoasinduction\_lem2  $\Rightarrow$  hoasinduction\_lem3b

hoasinduction\_lem3b\_lthm is an abbreviation defined by

hoasinduction\_lem3b

hoasinduction\_lem3bb is an abbreviation defined by

$(\forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o} F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}} M_{\text{subst}} A_{\text{term}} N_{\text{subst}}. \text{sub} (F M A) N = F (\text{comp } M N) (\text{sub } A N)) \Rightarrow \forall B_{\text{term}}$

Same formula in array format:

$(\forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o} F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}}$   
 $M_{\text{subst}} A_{\text{term}} N_{\text{subst}}. \text{sub} (F M A) N = F (\text{comp } M N) (\text{sub } A N))$   
 $\Rightarrow \forall B_{\text{term}}. (\exists F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}}. \text{sub } B (\text{push one}$   
 $\text{sh})) = F \text{ sh one}$

hoasinduction\_lem3bb\_lthm is an abbreviation defined by

axvarid  $\Rightarrow$  axvarshift  $\Rightarrow$  axclos  $\Rightarrow$  axmap  $\Rightarrow$  hoasinduction\_lem3bb

hoasinduction is an abbreviation defined by

$\forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o}. (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}} K_{\text{subst}}. P M A (\text{comp } K N) \Rightarrow P (\text{comp } M K) (\text{sub } A K) N) \Rightarrow (\forall M_{\text{subst}}$

Same formula in array format:

$\forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o}. (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}}$   
 $K_{\text{subst}}. P M A (\text{comp } K N) \Rightarrow P (\text{comp } M K) (\text{sub } A K) N) \Rightarrow (\forall M_{\text{subst}} A_{\text{term}}$   
 $N_{\text{subst}} K_{\text{subst}}. P (\text{comp } M K) (\text{sub } A K) N \Rightarrow P M A (\text{comp } K N)) \Rightarrow (\forall A_{\text{term}}$   
 $. \text{hoasvar id } A \text{ id} \Rightarrow P \text{ id } A \text{ id}) \Rightarrow (\forall A_{\text{term}} B_{\text{term}}. P \text{ id } A \text{ id} \Rightarrow P \text{ id } B \text{ id}$   
 $\Rightarrow P \text{ id } (\text{hoasap id } A \text{ id } B) \text{ id}) \Rightarrow (\forall F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}}. (\forall$   
 $M_{\text{subst}} A_{\text{term}} N_{\text{subst}}. \text{sub} (F M A) N = F (\text{comp } M N) (\text{sub } A N))$   
 $\Rightarrow (\forall A_{\text{term}}. P \text{ id } A \text{ id} \Rightarrow P \text{ id } (F \text{ id } A) \text{ id}) \Rightarrow P \text{ id } (\text{hoaslam id } \lambda M_{\text{subst}}$   
 $A_{\text{term}}. F M A) \text{ id}) \Rightarrow \forall A_{\text{term}}. P \text{ id } A \text{ id}$

`hoasinduction_gthm` is an abbreviation defined by

`axapp  $\Rightarrow$  axvarcons  $\Rightarrow$  axvarid  $\Rightarrow$  axabs  $\Rightarrow$  axclos  $\Rightarrow$  axidl  $\Rightarrow$  axshiftcons  $\Rightarrow$  axassoc  $\Rightarrow$  axmap  $\Rightarrow$  axidr`

*Same formula in array format:*

`axapp  $\Rightarrow$  axvarcons  $\Rightarrow$  axvarid  $\Rightarrow$  axabs  $\Rightarrow$  axclos  $\Rightarrow$  axidl  $\Rightarrow$  axshiftcons  $\Rightarrow$  axassoc  
 $\Rightarrow$  axmap  $\Rightarrow$  axidr  $\Rightarrow$  axvarshift  $\Rightarrow$  axscons  $\Rightarrow$  ulamvar1  $\Rightarrow$  ulamvarsh  $\Rightarrow$  ulamvarind  
 $\Rightarrow$  apinj1  $\Rightarrow$  apinj2  $\Rightarrow$  laminj  $\Rightarrow$  shinj  $\Rightarrow$  lamnotap  $\Rightarrow$  apnotvar  $\Rightarrow$  lamnotvar  $\Rightarrow$  induction  
 $\Rightarrow$  pushprop  $\Rightarrow$  induction2lem  $\Rightarrow$  induction2  $\Rightarrow$  substmonoid  $\Rightarrow$  termmset  $\Rightarrow$  hoasapinj1  
 $\Rightarrow$  hoasapinj2  $\Rightarrow$  hoaslaminj  $\Rightarrow$  hoaslamnotap  $\Rightarrow$  hoaslamnotvar  $\Rightarrow$  hoasapnotvar  
 $\Rightarrow$  hoasinduction_lem1  $\Rightarrow$  hoasinduction_lem2  $\Rightarrow$  hoasinduction_lem3  $\Rightarrow$   
hoasinduction`

`hoasinduction_lthm` is an abbreviation defined by

`induction2  $\Rightarrow$  hoasinduction_lem1  $\Rightarrow$  hoasinduction_lem2  $\Rightarrow$  hoasinduction_lem3  $\Rightarrow$  hoasinduction`

*Same formula in array format:*

`induction2  $\Rightarrow$  hoasinduction_lem1  $\Rightarrow$  hoasinduction_lem2  $\Rightarrow$  hoasinduction_lem3  
 $\Rightarrow$  hoasinduction`

`hoasinduction_lthm` is an abbreviation defined by

`hoasinduction_lem0  $\Rightarrow$  induction2  $\Rightarrow$  axvarid  $\Rightarrow$  hoasinduction_lem3v2a  $\Rightarrow$  hoasinduction`

*Same formula in array format:*

`hoasinduction_lem0  $\Rightarrow$  induction2  $\Rightarrow$  axvarid  $\Rightarrow$  hoasinduction_lem3v2a  
 $\Rightarrow$  hoasinduction`

`hoasinduction_no_psi_cond` is an abbreviation defined by

$\forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o}. (\forall A_{\text{term}} B_{\text{term}}. P \text{ id } A \text{ id} \Rightarrow P \text{ id } B \text{ id} \Rightarrow P \text{ id } (\text{hoasap id } A \text{ id } B) \text{ id}) \Rightarrow (\forall F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}}$

*Same formula in array format:*

$\forall P_{\text{subst} \rightarrow \text{term} \rightarrow \text{subst} \rightarrow o}. (\forall A_{\text{term}} B_{\text{term}}. P$   
 $\text{ id } A \text{ id} \Rightarrow P \text{ id } B \text{ id} \Rightarrow P \text{ id } (\text{hoasap id } A \text{ id } B) \text{ id}) \Rightarrow (\forall F_{\text{subst} \rightarrow \text{term} \rightarrow \text{term}}$   
 $. (\forall M_{\text{subst}} A_{\text{term}} N_{\text{subst}}. \text{sub } (F M A) N = F (\text{comp } M N) (\text{sub } A$   
 $N)) \Rightarrow (\forall A_{\text{term}}. P \text{ id } A \text{ id} \Rightarrow P \text{ id } (F \text{ id } A) \text{ id}) \Rightarrow P \text{ id } (\text{hoaslam id } \lambda M_{\text{subst}}$   
 $A_{\text{term}}. F M A) \text{ id}) \Rightarrow \forall A_{\text{term}}. P \text{ id } A \text{ id}$

`hoasinduction_no_psi_cond_lthm` is an abbreviation defined by

`hoasinduction_lem0  $\Rightarrow$  induction2  $\Rightarrow$  axvarid  $\Rightarrow$  hoasinduction_lem3v2a  $\Rightarrow$  hoasinduction_no_psi_cond`

*Same formula in array format:*

`hoasinduction_lem0  $\Rightarrow$  induction2  $\Rightarrow$  axvarid  $\Rightarrow$  hoasinduction_lem3v2a  
 $\Rightarrow$  hoasinduction_no_psi_cond`