Generating Infrastructural Code for Terms with Binders using MetaCoq

Bachelor Talk 2

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Motivation

Problem: Prove Metatheorems of Languages Modelled in Coq

- How to model binders and substitution

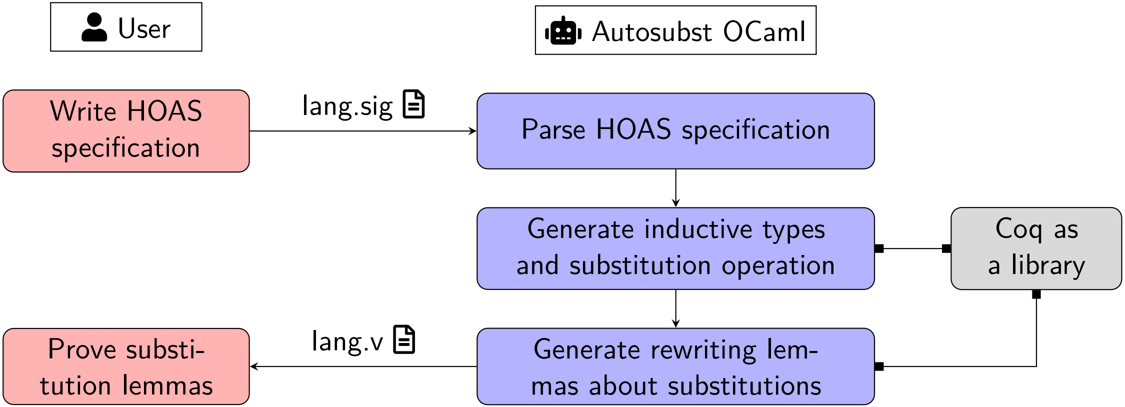
  $(\lambda x.t)v \succ_\beta t[x \mapsto v]$
- How to solve subtitution equations

  $\mathtt{s}\,[\sigma] \stackrel{?}{=} \mathtt{t}\,[\tau]$

Solution: Autosubst (Dissertation of Kathrin Stark [Stark, 2019])

- De Bruijn indices
- Based on sigma calculus [Abadi et al., 1991]
- Provides `asimpl` tactic to solve substitution equations

Workflow: Autosubst OCaml



Code Generation

- Variations of old lemmas supporting funext-free `asimpl` are generated
- Some original lemmas are optionally generated

Automation Generation

- Tactics can be constructed with tactic AST from Coq implementation (but Ltac commands can not)

```
let tac = repeat_ (first_ [ progress_ (setoid_rewrite_ asimpl_rewrite_lemma)
                          ; progress_ (unfold_ asimpl_unfold_functions)
                          ; progress_ (cbn_ asimpl_cbn_functions)
                          ... ]

Ltac asimpl := ...
```

- Typeclasses and instances can be constructed from the command & term ASTs

Code Generation

- Basic lemmas are generated
  (unscoped, functor-less and non-variadic syntax)
- wellscoped, functor and variadic syntax are straightforward extensions

Problems

- Implicit arguments
- Shadowing
- Recursive functions
- De Buijn indices

Implicit Arguments

Problem

Which arguments are implicit is not part of MetaCoq AST

Workaround

Pass "holes" (underscores in concrete syntax)

```
tmTypedDefinition "myList" hole (tApp <% @cons %> [hole; <% 0 %>; <% [] %>])
(* ⇒ myList : ?T := cons ?T0 0 [] *)
(* ⇒ mylist : list ℕ := [0] *)
```

Recursive Functions

Problem: Porting Recursive Functions to MetaCoq

Are all 23 recursive functions from OCaml terminating and implementable in Coq?

Answer: Yes

- Most are structurally recursive helper functions on lists
- Some use recursion nested in lists like rose trees
- One uses well-founded recursion with an agenda argument can be reformulated to use a fold

De Bruin Indices

Problem: Programming with De Bruin Indices is Hard

```
Fixpoint even (n: ℕ) :=
  match n with
  | S n ⇒ negb (odd n) | ...
with odd (n: ℕ) :=
  match n with
  | S n ⇒ negb (even n) | ...
```

```
tFix [
  tLam (tCase 0
    [ tLam (negb (2 0)); ... ]);
  tLam (tCase 0
    [ tLam (negb (3 0)); ... ])
  ]
```

Solution: Environments

Function `env : string` $\to \mathbb{N}$ that is updated when constructing a term below a binder

De Bruin Indices

Problem: Managing Environments is Hard

- Need to know the context before constructing a term

```
let smallerTerm = tApp (env "even") [env "n"] in
let t = buildBiggerTerm smallerTerm in
```

- Monadic functions are pervasive and you have to worry about order of execution

```
let mSmallerTerm = mApp (mEnv "even") [mEnv "n"] in
let t = mBuildBiggerTerm mSmallerTerm in
```

Solution: Custom AST with Named Variables

Translate the named variables to deBruijn indices after the whole term is built

asimpl With Setoid Rewriting

```
Lemma extequal : ∀ f g x, f x = g x.
```

Goal: Solve a Substitution Equation

```
∀ (s t: tm) f g h,
    s[t .: (h >> f)] = s[t .: (h >> g))].
```

Need morphisms for instantiation, scons and function composition†

```
Instance subst_morphism :
  Proper (pointwise_relation _ eq ⇒ eq ⇒ eq) (@subst_tm).
```

asimpl With Setoid Rewriting

Problems

- Setoid rewrite requires exact match (before typeclass resolution begins)

```
H : ∀ x, f x = g x

s[h >> f] = s[h >> g] (* Tactic failure: nothing to rewrite *)
s[fun x ⇒ f (h x)] = s[fun x ⇒ g (h x)]
```

- Morphisms are hard to get right
  Need one for all user-defined types with term indices (e.g. $\Gamma \vdash s[\sigma] : t$)
  even harder if language has nested recursion (e.g. record types)
- Slower

Allfv Lemmas

Existing infrastructure works well for this kind of new lemmas
Handle variable case, combination of recursive calls and lifting

```
Fixpoint subst (σ : ℕ → tm) (s : tm) :=
  match s with
| var s0 ⇒ σ s0
| app s0 s1 ⇒
      app (subst σ s0) (subst σ s1)
| lam s0 s1 ⇒ lam s0 (subst (⇑ σ) s1)
  end.
```

```
Fixpoint allfv (p: ℕ → ℙ) (s: tm) :=
  match s with
  | var x ⇒ p x
  | app s0 s1 ⇒
        allfv p s0 ∧ allfv p s1
  | lam s0 s1 ⇒ True ∧ allfv (⇑ p) s1
  end.
```

Code Statistics

LoC

	Haskell	OCaml	MetaCoq
code	2636	3285	2828
comments	310	437	-

Timings

asimpl

Comparing compilation times of a large case study (containing a.o. POPLmark[Aydemir et al., 2005])

functional extensionality	setoid-rewriting
111.7 seconds	412.0 seconds

Bugfixes

- Original Autosubst
  - Some printed notations
  - Unparseable substitution operation generated
  - Missing `{struct s}` annotation caused slowdonws
- Coq
  - Printing of "Existing Instances" command

Feature Table

	Autosubst OCaml	Autosubst MetaCoq	New `asimpl`
done	parsing basic lemmas lemmas for new `asimpl` tactics	parsing basic lemmas†	define lemmas morphisms proof-of-concept
todo	allfv lemmas full documentation publish	allfv lemmas full documentation publish †syntax extensions lemmas for new `asimpl` tactics	fix bugs

Maybe Todo

Faster PoC for `asimpl`, traced syntax, Autosubst webservice

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Shadowing

Problem: Shadow existing constants

When dynamically defining new constants from a meta-program

```
Inductive ty := ... | all : ty → ty.

(* all : reductionStrategy *)
tmUnquoteInductive "tm" (Some all) ind;;
(* all : ty → ty *)
tmDefinition "mydef" (Some all) term;; (* fails *)
```

Solution

Put user generated code into a module

Custom AST

```
Inductive term :=
| tRel : ℕ → term
| tProd : string → term → term → term
| tLambda : string → term → term → term
| tApp : term → term → term
| ...
```

```
Inductive nterm :=
| nRef : string → nterm
| nTerm : term → nterm
| nProd : string → nterm → nterm →
    nterm
| nLambda : string → nterm → nterm →
    nterm
| nApp : nterm → nterm → nterm
| ...
```

Faster Alternative to Setoid Rewriting

Do Setoid Rewriting Backwards

- setoid-rewriting: given an equality, find a path of morphisms that lead to being able to rewrite with that equality
- idea: because our rewriting is pretty regular, start applying morphisms as long as subterms are not equal and apply the rewrite lemmas if we can't decompose terms further
- works well on subsitution equations $s[\sigma] \stackrel{?}{=} t[\tau]$
- does not work on normalizing single terms $s[\sigma]$

Allfv use cases

- Closedness check with constant $\bot$ predicate
- Check if a term is wellscoped
- If type function instead of predicate, collect free variables in list
- Prove two substitutions equal if they agree only on the free variables

Allfv Lemmas

```
Fixpoint idSubst (σ : ℕ → tm)
  (Eq : ∀ x, σ x = var x) (s : tm) :
    subst σ s = s :=
  match s with
  | var s0 ⇒ Eq s0
  | app s0 s1 ⇒
      congr_app (idSubst σ Eq s0)
                (idSubst σ Eq s1)
  | lam s0 s1 ⇒
      congr_lam s0
          (idSubst (⇑ σ) (⇑ Eq) s1)
  end.
```

```
Fixpoint allfv_triv (p: ℕ → ℙ)
  (H: ∀ x, p x) (s: tm) :
    allfv p s :=
  match s with
  | var s0 ⇒ H s0
  | app s0 s1 ⇒
      conj (allfv_triv p H s0)
           (allfv_triv p H s1)
  | lam s0 s1 ⇒
      conj I
        (allfv_triv (⇑ p) (⇑ H) s1)
  end.
```