

Singular Euler-Maclaurin Expansion in Rocq

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M2 Internship Proposal

Contact: Feel free to contact any of the internship supervisors based in France.

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Location: The internship can be hosted at Inria Lyon, Inria Paris or Inria Rennes (Nantes site). Andreas Buchheit, based in Saarbrücken, Germany, will act as remote supervisor.

Requirements: Required are proficiency in using Rocq as well as a basic understanding of analysis.

Context: The Singular Euler-Maclaurin expansion (SEM) is a relatively recent generalisation [1, 2] of a more than 300-years-old summation formula developed by Euler and Maclaurin. While the original formula allows to efficiently compute large sums by means of integration and applies to everywhere continuous functions, the SEM extends the method to functions with singularities, for instance those commonly used to describe particle interactions and other physical forces. The derived algorithm of the SEM outperforms previous numerical approximations of these forces and therefore found applications in several domains of physical research [3], even receiving considerable attention in the media.¹²

Goals: In the context of the growing interest in formalised mathematics and formally verified numerical algorithms, the goal of this internship is to formalise the SEM in Rocq’s MathComp Analysis library [4, 5]. Intermediate milestones are a formalisation of the original Euler-Maclaurin expansion as well as the SEM in the one-dimensional case. Targeting a more long-term perspective, the project thereby lays the foundation for a systematic use of proof assistants in computational physics.

References

- [1] Andreas A Buchheit and Torsten Keßler. Singular euler–maclaurin expansion on multidimensional lattices. *Nonlinearity*, 35(7):3706, 2022.
- [2] Andreas A Buchheit and Torsten Keßler. On the efficient computation of large scale singular sums with applications to long-range forces in crystal lattices. *Journal of Scientific Computing*, 90(1):53, 2022.
- [3] Andreas A Buchheit, Torsten Keßler, Peter K Schuhmacher, and Benedikt Fauseweh. Exact continuum representation of long-range interacting systems and emerging exotic phases in unconventional superconductors. *Physical Review Research*, 5(4):043065, 2023.
- [4] Assia Mahboubi and Enrico Tassi. Mathematical components. *Online book*, 2021.
- [5] Reynald Affeldt. An introduction to mathcomp-analysis. 2023.

¹<https://blog.wolfram.com/2021/06/30/the-singular-euler-maclaurin-expansion-a-new-twist-to-a-centuries-old-problem/>

²<https://www.spektrum.de/magazin/langreichweitige-kraefte-verallgemeinerung-der-euler-maclaurin-formel/2030575>