1 Personal Information

Dominik Kirst, MSCA-Postdoctoral Fellow at Picube Project-Team within Inria Paris

2 Research Achievements

- D. Kirst and G. Smolka. Categoricity results and large model constructions for second-order ZF in dependent type theory. *Journal of Automated Reasoning*, 63(2):415–438, 2019.

  Impact Our mechanised representation of set theory in type theory influenced other mechanisations of set theory [6, 49, 34] and the included inductive conception of ordinal numbers was reused for an extension of the Iris framework for program verification [48].

  Role I took the lead in conceptualising, developing, mechanising, writing, and presenting the contents of this journal article and the included conference papers, while my PhD supervisor Gert Smolka provided feedback and parts of the informal text.

- Y. Forster, D. Kirst, and G. Smolka. On synthetic undecidability in Coq, with an application to the Entscheidungsproblem. In *International Conference on Certified Programs and Proofs*. ACM, 2019.

  Impact With currently 85 citations according to Google Scholar this is my most-cited paper, mostly due to being the canonical reference point for our substantial line of work concerning synthetic undecidability and computability theory in general.

  Role Together with my PhD colleague Yannick Forster I shared the responsibilities for conceptualising, developing, and mechanising the contents of this paper, while I took the lead in writing and our PhD supervisor Gert Smolka provided feedback and parts of the informal text.

- Y. Forster, D. Kirst, and D. Wehr. Completeness theorems for first-order logic analysed in constructive type theory: Extended version. *Journal of Logic and Computation*, 31(1):112–151, 2021.

  Impact The work formed the base of my ongoing interest in the constructive content of completeness theorems [38, 22, 52, 45, 35], posed two open questions that were resolved in [23] and [8], and our insights and mechanisation techniques were picked up by other researchers [4, 42, 12, 21, 2].

  Role This work is based on Dominik Wehr's Bachelor's project that I co-advised with Yannick Forster, providing the initial idea and framework. During the project I contributed several insights, parts of the Coq development, as well as most of the writing and the conference presentation.

- D. Kirst and M. Hermes. Synthetic undecidability and incompleteness of first-order axiom systems in Coq. In *International Conference on Interactive Theorem Proving*. LIPIcs, 2021.

  Impact This paper and its extended journal version [29] uses the synthetic approach to undecidability developed in [17] to target Gödel's first incompleteness theorem, thereby opening a new angle in the investigation of this fundamental result of mathematical logic. As a direct follow-up work, it prepared the deeper study of the incompleteness phenomenon conducted in [33].

  Role This work is based on Marc Hermes' Master's project that I advised, providing the initial idea and framework. In this first project phase, I took the lead in developing the definitions, proofs, and mechanisation while I advised the student to work out some technical details.

- M. Hermes and D. Kirst. An analysis of Tennenbaum's theorem in constructive type theory. In *International Conference on Formal Structures for Computation and Deduction*. LIPIcs, 2022.

  Impact The paper received the best student paper award at FSCD'22 and was subsequently selected for extended publication in the LMCS journal [25]. The refactored mechanisation formed the basis for the Coq library of first-order logic [30] and some involved results were reused in later works.

  Role This work is based on Marc Hermes' Master's project that I advised, providing the initial idea and framework. Being on graduate level, the main project was mostly driven by the student and only directed via my feedback, consequentially I solely contributed to the informal parts of the paper.

- D. Kirst, J. Hostert, A. Dudenhefner, Y. Forster, M. Hermes, M. Koch, D. Larchey-Wendling, N. Mück, B. Peters, G. Smolka, and D. Wehr. A Coq library for mechanised first-order logic. In *Coq Workshop*, 2022.

  Impact This workshop abstract describes the Coq library for first-order logic, a collaborative project of 11 contributors, spanning more than 25.000 loc, and containing code of [17, 18, 19, 31, 32, 28, 26, 24, 33, 29], with the code of [45, 35, 37] in the process of being merged.

  Role I am the lead developer of the library, formed after consolidating several evolution steps from the corresponding underlying projects. The final design has been discussed and implemented together with Johannes Hostert, whom I supervised as an engineering intern for that purpose.

- D. Kirst and B. Peters. Gödel's theorem without tears - essential incompleteness in synthetic computability. In *31st EACSL Annual Conference on Computer Science Logic*. LIPIcs, 2023.

  Impact The work further contributes to the popularisation of the historically neglected fully computational explanation of Gödel's first incompleteness theorem, forming an accessible strategy for teaching that I was invited to turn into a course at the ANU Logic Summer School. On the technical side, it inspired follow-up projects on the related second incompleteness theorem and Löb's theorem.

  Role This work is based on Benjamin Peters' Bachelor's project that I advised, providing the initial idea and framework. During the project I contributed several insights, parts of the writing, and the conference presentation.

- Y. Forster, D. Kirst, and N. Mück. Oracle computability and Turing reducibility in the calculus of inductive constructions. In *Asian Symposium on Programming Languages and Systems*, pages 155–181. Springer, 2023.

  Impact The work settled a long-winded search for a synthetic rendering of oracle computability, thereby succeeding several failed attempts. Therefore, the paper opened up an ongoing line of work to investigate more advanced results about oracle computability, such as the Post's hierarchy theorem [16] and Post's problem [53].

  Role This work is based on Niklas Mück's Bachelor's project that I co-advised with Yannick Forster, providing the initial idea and framework. During the project I contributed several insights as well as parts of the writing.

- L. Cohen, Y. Forster, D. Kirst, B. da Rocha Paiva, and V. Rahli. Separating Markov's principles. In *39th Annual ACM/IEEE Symposium on Logic in Computer Science (LICS'24), July 8–11, 2024, Tallinn, Estonia*, 2024.

  Impact While being too recent to assess the impact on the research community, for my line of work in synthetic computability the result is crucial and already triggered follow-up work on a related logical axiom called the limited principle of omniscience [11]. Moreover, evidencing the potential recognition this paper might receive, I was invited to present the work at a CHoCoLa meeting in Lyon, a monthly nationwide seminar connecting researchers in logic and computation.

  Role The project idea arose from discussing an open problem with Liron Cohen and Vincent Rahli in the context of their models of type theory with choice sequences. During the project I contributed several insights as well as parts of the writing.

- D. Kirst and I. Shillito. Completeness of first-order bi-intuitionistic logic. In *33rd EACSL Annual Conference on Computer Science Logic (CSL 2025)*. LIPIcs, 2025.

  Impact While also being too recent to assess the impact on the research community, this paper settles a long-winded debate concerning the correctness of several alleged proofs for the completeness of first-order bi-intuitionistic logic in the literature and extends previous work with Ian Shillito [45].

  Role The project idea arose after Shillito identified several errors in existing proofs and we already settled the propositional fragment of the logic. During the project, conducted mostly as a research visit at the ANU in Australia, I contributed several insights as well as parts of the writing.

3 Scientific Proposal: Unified Reverse Mathematics

My proposed research project is centred around the programme of reverse mathematics, i.e. the fine analysis of the logical strength of foundational mathematical theorems, whose two dialects called classical and constructive reverse mathematics are well-known to be orthogonal and seemingly incompatible, while intrinsically connected and individually fruitful. With agnostic type-theoretic foundations and the systematic use of proof assistants, however, this rift can be overcome, providing a more general and reconciled understanding of logical foundations of mathematical research, and thus my long-term goal is to develop a formal framework for unified reverse mathematics.

3.1 Vision and State of the Art

The research programme of (classical) reverse mathematics [20] founded by Friedman in the 1970s is an ongoing effort concerned with the exact identification of the necessary logical principles underlying mathematical theorems. Next to the motivation of clarifying the role of subtle, at times debatable logical assumptions, other reasons to analyse theorems in the spirit of reverse mathematics are to observe their most general formulation and to gain insights about their internal logical structure. For these goals, the choice of the base theory is crucial: the weaker the base assumptions, the finer the lens for the analysis. So instead of using a rich system like axiomatic set theory, the mainstream foundation of mathematics, the most established base theories are the so-called Big Five, fragments of second-order arithmetic containing set existence axioms of ascending strength [46].

On top of the arithmetical language, the Big Five share a common logical core, namely so-called classical logic including the law of excluded middle (LEM), stating that every proposition P is either true or false ($\forall P.\, P \vee \neg P$). If LEM is left out, however, then one obtains so-called constructive logic, a variant unveiling logical principles weaker than LEM like Markov's principle, weak excluded middle, or double-negation shift. In contrast to classical proofs, constructive proofs bear explicit computational information, for instance every constructive proof of an existential quantification $\exists x.\, P(x)$ provides a guarantee that a concrete witness x of P can be computed. While modern mainstream mathematics continues to be developed in classical logic, there is a growing community developing applications of constructive mathematics, especially in domains related to computer science [5].

By switching to constructive logic, the programme of reverse mathematics is extended into a further dimension: on top of the contribution of set existence axioms to a given theorem, also the role of sub-classical principles can be analysed [27]. Such observations become especially relevant if considered from the computational perspective: If it is provable constructively (i.e. without LEM), the computational interpretation can be easily read off. If it is equivalent to some sub-classical principle, any extension of computation with additional primitives realising the principle automatically also realises the theorem. Today, there is increasing interest in constructive reverse mathematics but many foundational theorems are yet to be fully analysed regarding their computational content [13].

In the current state of affairs, the communities of classical and constructive reverse mathematics, while in principle following the same quest for a fine-grained logical understanding of foundational mathematical theorems, are relatively disconnected. Not only do they work in different formal foundations and focus on different dimensions of logical strength, but also are their results communicated mostly targeting their own audience, leaving room for misunderstandings, for instances visible at the seemingly conflicting results about the completeness theorem of first-order logic [39, 51, 41, 46]. Similarly as I have already done in that case [19, 23], I plan to develop unified classical and constructive perspectives on other areas of interest, bridging the gap between the two communities via translation of motivating problems and transfer of technical insights.

There are two factors making this goal achievable only now. First, the emergence of constructive type theories over the recent decades, such as the calculus of inductive constructions (CIC)[10, 43], led to expressive but minimalistic foundational systems that are well-suited to unify the many diverging base systems of classical and constructive reverse mathematics. Secondly, constructive type theories are readily implemented in so-called proof assistants, that is, computer programs that allow the user to mechanise mathematical results, i.e. to write definitions, theorems, and proofs in a machine-checkable format, providing correctness guarantees especially valuabe for a domain of high technical complexity like unified reverse mathematics as envisioned in this project.

So going one step further than a unified framework for classical and constructive reverse mathematics, as modern technology from interactive theorem proving enables the systematic codification of large bodies of mathematical results, I plan to develop a comprehensive formal library for reverse mathematics providing a definite reference point for any related work in the field. The preferred proof assistant for this work is Coq [50] (soon to be renamed into Rocq), as it is currently the only mature and agnostic implementation of CIC. However, before implementing a reasonable Coq library, several design decisions regarding the representation of the base systems and the considered results need to be settled in advance. The former includes the decision whether CIC can be used itself with the addition of axioms to represent the respective base systems (a shallow embedding) or if they need to be described syntactically as a concrete inductive structure in CIC (a deep embedding). The latter includes the careful distinction of the constructively separate but classically equivalent formulation of the target results. Apart from the use to codify existing results, experience shows that using proof assistants supports already the development of new results by providing guidance and automation for the researcher, especially for intricate technical domains like reverse-engineering of logical assumptions.

An additional dimension of my research project will be the construction of suitable models of the involved base systems like CIC and other constructive type theories, in order to analyse the status of newly identified logical principles. Next to the technical aspect of studying models for custom purposes, my more foundational goal is to develop tools easing such model constructions in general, such that non-experts in the categorical semantics of type theory still have access to semantical techniques. With several working groups within my research network I am planning to investigate models based on choice sequences [9], presheaf toposes [1], and effectful realisability [7] to study this and other fragments of well-known logical principles that we already found in our work on the meta-theory of first-order logic. Another pressing need solvable by such semantical arguments is the justification of the mutual consistency of classical axioms like the law of excluded middle with computational axioms like Church's thesis [40], which is routinely used for synthetic computability.

This concrete motivation, however, is part of a more general research programme I plan to push forward over the next years: investigate ways how the common category-theoretic semantics of type theory, requiring lots of formal prerequisites, can be circumvented via simpler constructions rather amounting to internal syntactic translation than to external denotational reinterpretations (cf. [3, 44]). The great advantage of such an approach is that researchers with a background in the syntactical side are not necessarily forced to get acquainted with the semantical details to analyse properties like consistency and independence of axioms. As a first step in that direction, I work on a unified framework for effectful realisability, treated as a fully syntactic translation while still providing a powerful toolset for the construction of suitable models.

3.2 Grand Challenge and Scientific Objectives

The long-term outcome of this effort is a formal framework accommodating the results observed by both communities of classical and constructive reverse mathematics in a mutually understandable way. For this to be achievable, type theories like CIC provide an ideal setting as their agnostic logical foundation allows to simulate both fully constructive and fully classical systems, so a primary task is to reframe the bodies of already obtained results for other systems like constructive and classical arithmetics in terms of CIC. Altogether, this project is of foundational interest, as it sheds light on the deep connections of computation and logic, ambitious, as it involves many mathematical and social challenges, but also realistic, as my intersecting research networks and I together have the necessary expertise in the relevant, often disconnected fields.

A first main challenge of the proposed project is the sheer technical complexity and richness of reverse mathematical results. Especially in the constructive case, the observable logical strength of analysed theorems depends on many subtle factors in the concrete formulation [23], often giving rise to a magnitude of classically equivalent but constructively distinguishable logical principles [4].

A second main challenge is the social phenomenon that the two targeted communities are completely disconnected, while in principle sharing related motivations. As a consequence, two different bodies of literature, all coming with their own traditions and interests need to be reconciled.

The following are the main objectives of the proposed research project:

Objective 1 is the reformulation of the existing bodies of classical and constructive reverse mathematics within a suitable unified system. This mostly involves the identification of the suitable system, which is anticipated to be a fragment of CIC with controlled behaviour and therefore embodying the most agnostic foundation.

Objective 2 is the use of the obtained unification to transfer results from one setting into the other. This presupposes the understanding how the seemingly orthogonal axes of logical strength in both dialects are actually connected and therefore observations in the one dimension related to observations in the other dimension.

Objective 3 is the development of a proof assistant allowing to smoothly simulate the different base systems at play and providing realistic tool support. This will most likely be an adaptation of the Coq proof assistant implementing said fragment of CIC and will incorporate different modes to simulate the different base systems.

Objective 4 is the implementation of a comprehensive formal library of unified reverse mathematics. This library and the additionally implemented tool support will then serve as basis for external users aiming to mechanise their ongoing research.

Objective 5 is to clarify the logical foundations of the involved foundational systems by constructing respective models. This includes the aim to provide simplifying interfaces for model constructions for specific purposes, circumventing the need for purely semantical investigations.

3.3 Risks and Impact

Given its technical complexity and interdisciplinary conception, the proposed project has high risks, requiring expertise in all connected fields. However, it is also expected to have a correspondingly high impact in form of a substantial change of mathematical practice, disseminated at top tier conferences and journals.

In principle, all scientific objectives of the project carry their own risks. For objectives 1 and 2, there is the potential that CIC or its fragments do not suffice to cover and transfer all the envisioned material and therefore an alternative needs to be developed. For objectives 3 and 4, adapting Coq to the needs of the project and developing tool support could involve unexpected engineering challenges. For objective 5, there is the mathematical risk that the search for suitable models, separating the analysed logical principles, remains without success. All of this risks can be mitigated through my special role at the intersection of the research communities targeting reverse mathematics, computer mechanisation, and programming languages semantics, bringing together the necessary expertise to overcome the potential dangers.

The results of this research project will be disseminated at top-tier scientific venues in continuation of my previous track record, therefore combining theory-oriented venues like CSL, FSCD, and LICS with mechanisation-focused venues like CPP, ITP, and JAR.

Its direct main impact will be of theoretical nature, namely the long-sought reconciliation of classical and constructive reverse mathematics, deepening the computational understanding of the logical foundations of mathematics. This in particular brings closure to Friedman's original initiation of the reverse mathematics programme and allows to have the two emerged scientific sub-communities profit from each others work.

Additionally, the project will have technological impact via its development of a well-suited proof assistant and accompanying formal library. Not only will this serve the purpose of integrating further branches of mathematics in the general vision of having comprehensive formal libraries of machine-checked proofs, but also the existing meta-programming tools like Elpi [14] and Metacoq [47] will be extended to accommodate the needs of the proposed project.

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