Formal Verification of a Family of Spilling Algorithms

Second Bachelor Seminar Talk

Julian Rosemann
Advisors: Prof. Gert Smolka, Sigurd Schneider

Saarland University
Department of Computer Science

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Recap: Spilling

code	r_1	r_2	r_3
…	x	y	
let z = x + y in	x	y	z
if z ≥ y then	x	y	z
x + z			
else			
z			

- x, y, z variables in register
- X, Y, Z variables in memory
- spill: let X=x in
- load: let x=X in
- memory variables only in loads & spills

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IL

$$
\begin{aligned}
s, t ::= \ & \text{let } x = e \text{ in } s \\
\mid \ & \text{if } e \text{ then } s \text{ else } t \\
\mid \ & e \\
\mid \ & \text{fun } f\ \overline{x} = s \text{ in } t \\
\mid \ & f\ \overline{x}
\end{aligned}
$$

Formally described in *Schneider, Smolka, and Hack, "A First-Order Functional Intermediate Language for Verified Compilers", 2015*

Outline

Representation of Spilling

code	S	L
...		
let X = x in		
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Representation of Spilling

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...		
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x	{}	{X}
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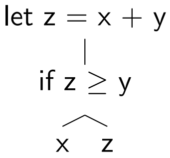

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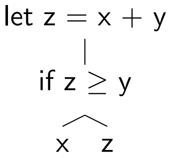

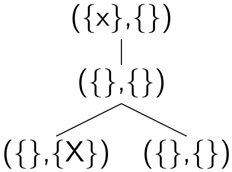

Representation of Spilling

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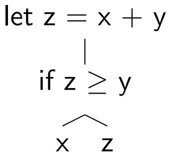

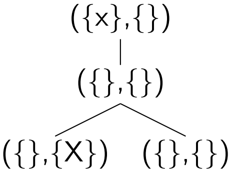

\+ additional information at function declaration and application

From a spilling to a spilled program

$$\texttt{do\_spill} : \ \texttt{stmt} \rightarrow \texttt{spilling} \rightarrow \texttt{stmt}$$

From a spilling to a spilled program

`do_spill : stmt → spilling → stmt`

$$(s, (\underbrace{\{x_1, ..., x_n\}}_{\text{spills}}, \underbrace{\{Y_1, ... Y_m\}}_{\text{loads}})) \mapsto \begin{array}{c} let\ X_1 = x_1\ in \ldots let\ X_n = x_n\ in \\ let\ y_1 = Y_1\ in \ldots let\ y_m = Y_m\ in \\ s \end{array}$$

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$spill_k$ on s and $sl \Rightarrow sl$ is valid on s.

Inductive Correctness Predicate

$$(ZL, \Lambda); (R, M) \vdash \mathsf{spill_k}\ \ lv\ \ s : sl$$

ZL
Λ
$x \in R$
$x \in M$
k
lv
s
sl

Inductive Correctness Predicate

$$(ZL, \Lambda); (R, M) \vdash \mathsf{spill_k}\ \ lv\ s : sl$$

ZL	list of parameters of defined functions
Λ	
$x \in R$	
$x \in M$	
k	
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$$(ZL, \Lambda); (R, M) \vdash \mathsf{spill}_\mathsf{k} \ \ lv \ s : sl$$

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Λ	list of expected live variables at function heads
$x \in R$	
$x \in M$	
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$x \in R$	:⇔ current value is in a register
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k	register bound
lv	
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k	register bound
lv	liveness information
s	source program
sl	spill/load information

Inductive Correctness Predicate

- $(ZL, \Lambda); (R, M) \vdash \mathrm{spill_k}\ (LV \cdot lv_s)\ (\mathrm{let}\, x := e\, \mathrm{in}\, s) : (Sp, L, None) \cdot sl_s$
- $(ZL, \Lambda); (R, M) \vdash \mathrm{spill_k}\ (LV \cdot lv_s, lv_t)\ (\mathrm{if}\, e\, \mathrm{then}\, s\, \mathrm{else}\, t)$
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- $(ZL, \Lambda); (R, M) \vdash \mathrm{spill_k}\ \ LV\ \ e : (Sp, L, None)$
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In the first talk for `let`:

$$\frac{\begin{array}{cc} Sp \subseteq R & L \subseteq Sp \cup M \\ |(R \setminus K \cup L) \setminus K_x \cup \{x\}| \leq k & fv(e) \subseteq R \setminus K \cup L \\ |R \setminus K \cup L| \leq k & (ZL, \Lambda); ((R \setminus K \cup L) \setminus K_x \cup \{x\}, Sp \cup M) \vdash \mathsf{spill}_\mathsf{k}\ \ lv_s\ \ s : sl \end{array}}{(ZL, \Lambda); (R, M) \vdash \mathsf{spill}_\mathsf{k}\ (LV \cdot lv_s)\ (\mathsf{let}\, x := e\, \mathsf{in}\, s) : (Sp, L, None) \cdot sl}$$

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$$\frac{\begin{array}{l|l} L \subseteq M & \text{loaded variables available} \\ |R \setminus K \cup L| \leq k & \text{don't load too much} \\ (ZL, \Lambda); (R \setminus K \cup L, M) \vdash \text{spill}_\text{k}\ \ lv\ \ s : (\emptyset, \emptyset, rm) \cdot \_ & \text{induction} \end{array}}{(ZL, \Lambda); (R, M) \vdash \text{spill}_\text{k}\ \ lv\ \ s : (\emptyset, L, rm) \cdot \_}$$

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$$\frac{\begin{array}{ll} fv(e) \subseteq R & \text{variables available} \\ |R \setminus K_x \cup \{x\}| \leq k & \text{register bound afterwards} \\ (ZL, \Lambda); (R \setminus K_x \cup \{x\}, M) \vdash \mathsf{spill_k}\ \ lv_s\ \ s : sl_s & \text{induction} \end{array}}{(ZL, \Lambda); (R, M) \vdash \mathsf{spill_k}\ (LV \cdot lv_s)\ (\mathrm{let}\, x := e \,\mathrm{in}\, s) : (\emptyset, \emptyset, None) \cdot sl_s}$$

variables in register & register bound

$$\frac{Sp \subseteq R \qquad (ZL, \Lambda); (R, M \cup Sp) \vdash \mathsf{spill_k}\ \ lv\ \ s : (\emptyset, L, rm) \cdot \_}{(ZL, \Lambda); (R, M) \vdash \mathsf{spill_k}\ \ lv\ \ s : (Sp, L, rm) \cdot \_}$$

spilled variables available
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loaded variables available
don't load too much
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variables available
register bound afterwards
induction

variables in register & register bound

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loaded variables available
don't load too much
induction

$$\frac{\begin{array}{l} fv(e) \subseteq R \\ |R \setminus K_x \cup \{x\}| \leq k \\ (ZL, \Lambda); (R \setminus K_x \cup \{x\}, M) \vdash \mathsf{spill_k}\ \ lv_s\ \ s : sl_s \end{array}}{(ZL, \Lambda); (R, M) \vdash \mathsf{spill_k}\ (LV \cdot lv_s)\ (\mathsf{let}\, x := e\, \mathsf{in}\, s) : (\emptyset, \emptyset, None) \cdot sl_s}$$

variables available
register bound afterwards
induction

Reconstructing Liveness

Liveness after spilling needed by:

Reconstructing Liveness

Liveness after spilling needed by:

- proof of register bound *and*

Reconstructing Liveness

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Approach:

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Reconstructing Liveness

Liveness after spilling needed by:

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Approach:

- use original liveness algorithm *or*
- develop own algorithm using
  - original liveness *and*
  - register and memory states at function heads

Reconstructing Liveness

original algorithm:

new algorithm:

Reconstructing Liveness

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- runs in $\mathcal{O}(n^3)$

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- challenge: kill sets are not available computationally

Liveness

code	live variables
let y = z in if x $\geq$ 0 then x else z	

Liveness intuition: variable x is **live** in statement s if either

- its value is used in s

or

- x was defined in the preceding statement

Liveness

code	live variables
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if x ≥ 0	
then x	
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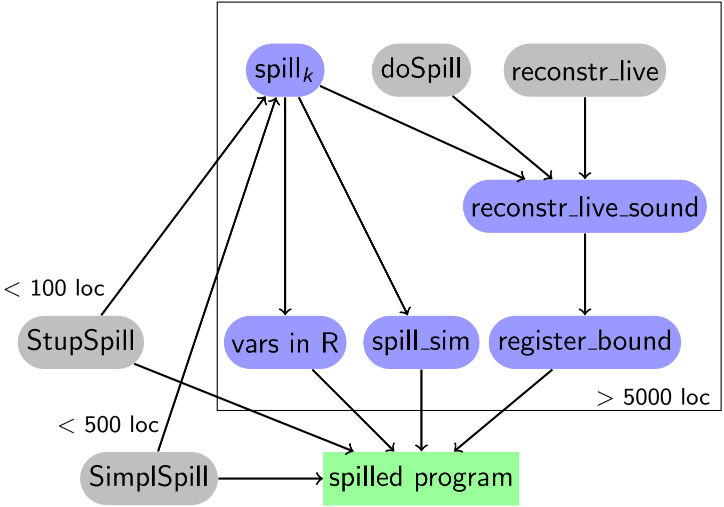
- spills are selected arbitrarily
- possible factorization: (unproven) oracle specifies priorities on selection of spills

Overview


spill$_k$
doSpill
reconstr_live
reconstr_live_sound
< 100 loc
StupSpill
vars in R
spill_sim
register_bound
> 5000 loc
< 500 loc
SimplSpill
spilled program

Conclusion

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