Formal Verification of Spilling Algorithms

Bachelor Talk

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Department of Computer Science

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Outline

Subproblems of Register Allocation

	r_1	r_2

```
let y  := 1  in
let z  := x  in


if z   >= w
then
  z
else

  w  + y
```

- Spilling: determine whether a variable is in the registers or in memory
- Register Assignment: determine in which register a variable resides
- Coalescing: reduce copy-instructions

Subproblems of Register Allocation

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```
let W  := w  in
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let z  := x  in
let Y  := y  in
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	r_1	r_2
	w	x
let W := w^1 in	w	⋮
let y^1 := 1 in	y	x
let z^2 := x^2 in	⋮	z
let Y := y^1 in	y	⋮
let w^1 := W in	w	⋮
if z^2 >= w^1	⋮	⋮
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`let W := w`^{1} `in`	w	⋮
`let y`^{1} `:= 1 in`	y	⋮
`let Y := y`^{1} `in`	y	⋮
`let w`^{1} `:= W in`	w	⋮
`if x`^{2} `>= w`^{1}	⋮	⋮
`then`		
`x`^{2}	⋮	x
`else`		
`let y`^{2} `:= Y in`	⋮	y
`w`^{1} `+ y`^{2}	w	y

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only possible in SSA

Global Register Allocation

- register assignment is independent of program point
- graph coloring algorithm *by Chaitin (1981)*

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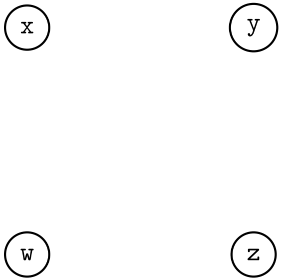

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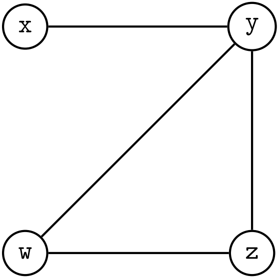

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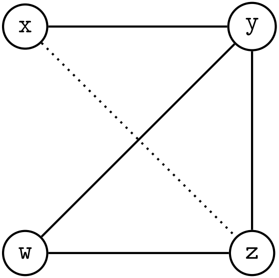

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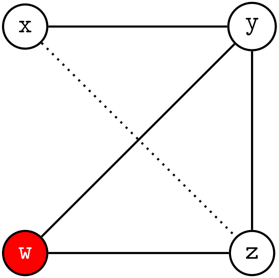

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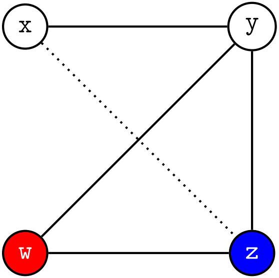

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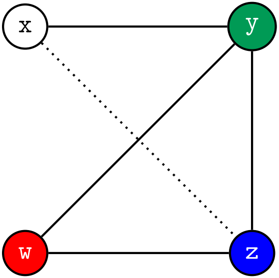

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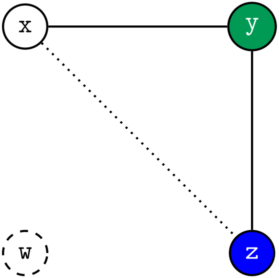

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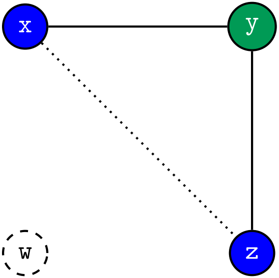

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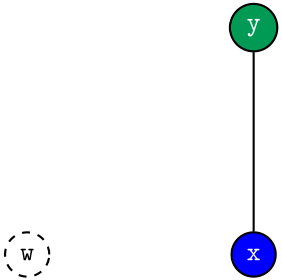
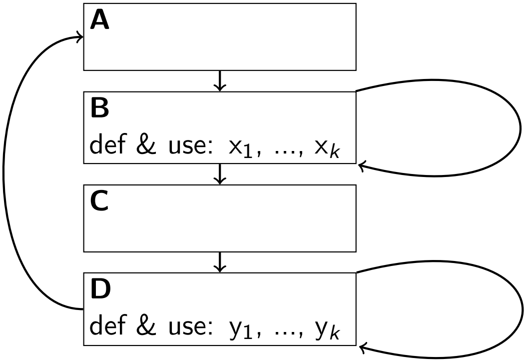


Problems with Global Register Allocation



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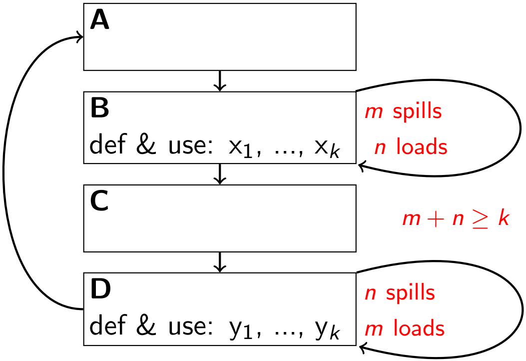

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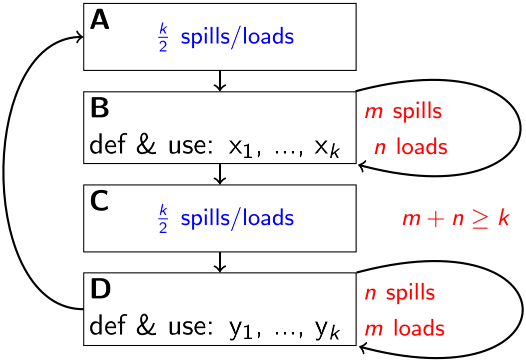
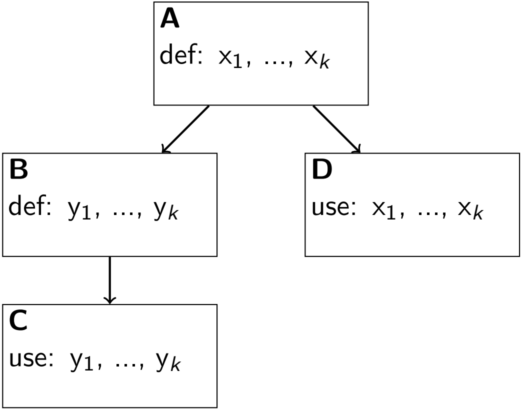


Linear Scan *by Poletto & Sarkar (1999)*



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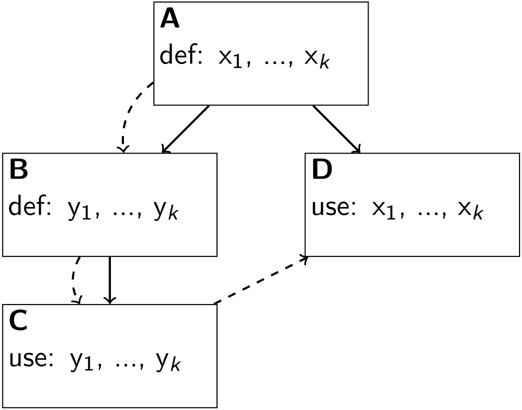

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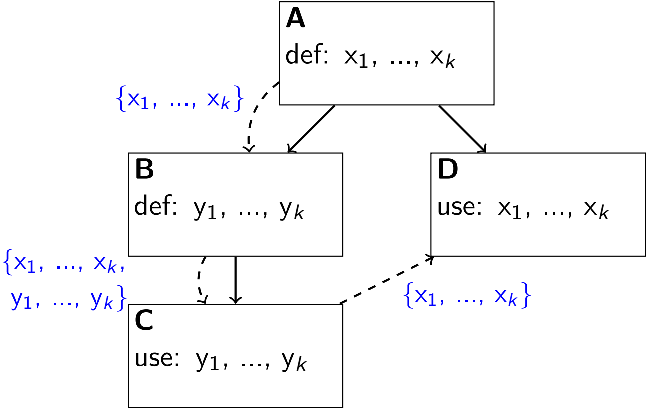

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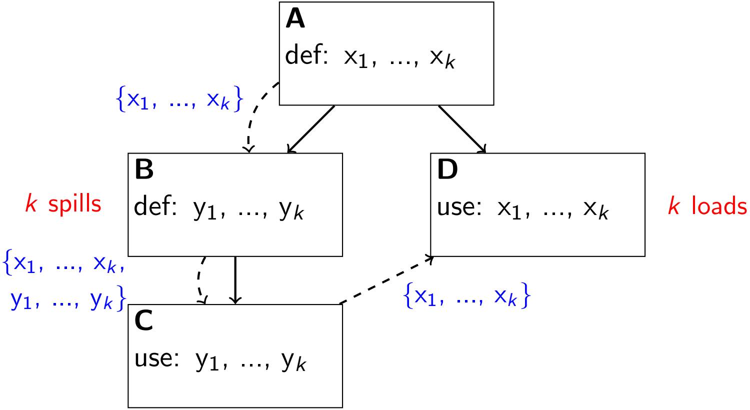
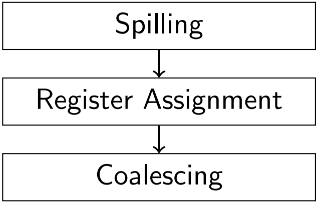


Register Allocation in SSA *by Hack et al. (2006)*



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Spilling
↓
Register Assignment
↓
Coalescing

- inference graphs of SSA-programs are chordal

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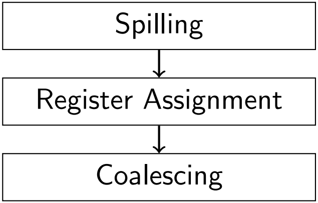

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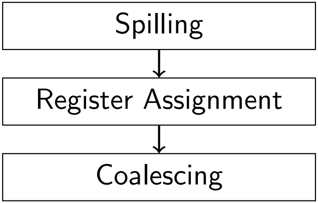
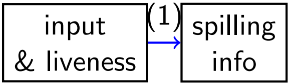


- inference graphs of SSA-programs are chordal
- efficient algorithm for register assignment
- coalescing using a heuristic

input
& liveness



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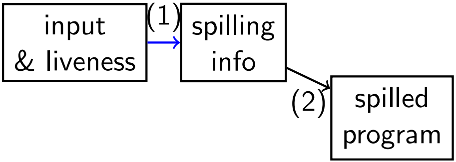

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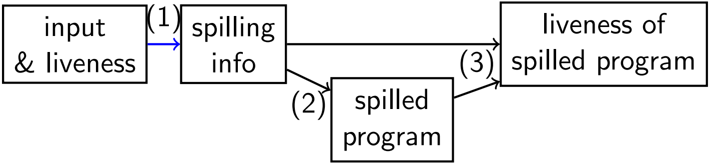

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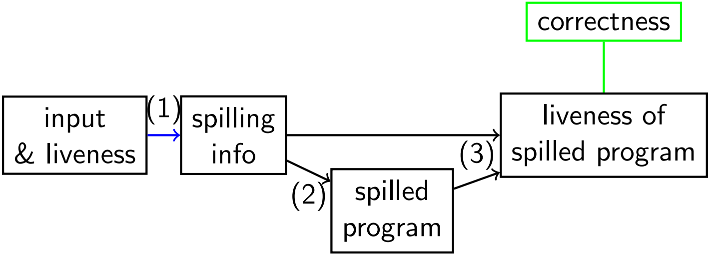

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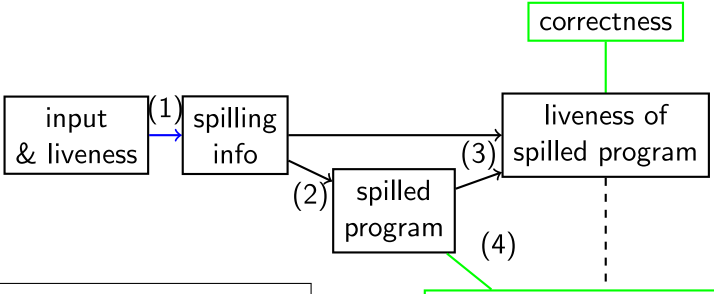

(1) spilling algorithm

(2) doSpill

(3) spilledLive

(4) correctness predicate

valid spilling

(a) variables in registers

(b) register bound

(c) program equivalence

Spilling Algorithms

StupSpill:

SimplSpill:

Spilling Algorithms

StupSpill:

- at any statement:

SimplSpill:

Spilling Algorithms

StupSpill:

- at any statement:
  - load everything

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Spilling Algorithms

StupSpill:

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SimplSpill:

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- specification of algorithm 70 l.
- verifying the predicate < 500 l.

Compute the Spilled Program

$$\text{doSpill}(s : (\underbrace{\{x_1, ..., x_n\}}_{\text{spills}}, \underbrace{\{y_1, ..., y_m\}}_{\text{loads}})) =$$

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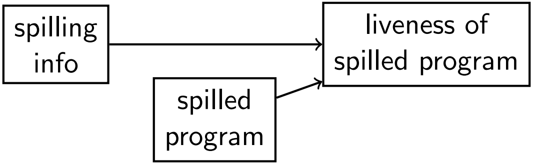
- `doSpill` is called recursively on every substatement
- the parameters are adjusted to the spilling information

Liveness of the spilled program



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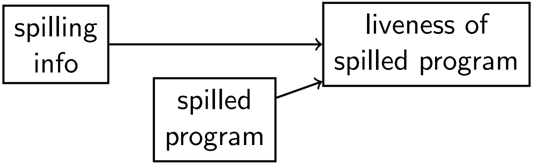

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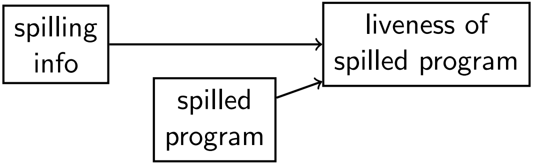

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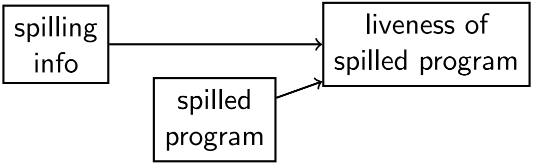

- spilling information includes liveness at function definitions
- construct liveness in one pass over the program
- proof of correctness was involving

Correctness Predicate

$$Z \mid \Sigma \mid R \mid M \vdash \mathbf{spill_k}\ \ s : (S, L, \_)$$

Z
Σ
$x \in R$
$x \in M$
k
s
S
L
$\_$

Correctness Predicate

$$Z \mid \Sigma \mid R \mid M \vdash \mathbf{spill_k}\ \ s : (S, L, \_)$$

Z	list of parameters of defined functions
Σ	list of expected live variables at function heads
$x \in R$	$:\Leftrightarrow$ current value is in a register
$x \in M$	$:\Leftrightarrow$ current value is in the memory
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k	register bound
s	source program
S	variables to be spilled
L	variables to be loaded
$\_$	additional information for functions

Soundness of the Predicate

Let s be renamed apart, $|R| \leq k$ and applications only have variables as arguments. If

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- at any program point the register contains at most k variables
- the translation preserves program equivalence

Verification of other Spilling Algorithms

e.g.: spilling algorithm *by Braun and Hack (2009)*:

Concepts of the algorithm: Verification:

Verification of other Spilling Algorithms

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Verification:

- verify the method computing distance to next-use
- keep track of register and memory state
- induction on the statement
- more involving than verification of SimplSpill, but similar

Translation Validation with Success Guarantee

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- iterate over program flow, liveness and spilling information:
  - if free variables of the expression are not in the registers, load them
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Translation Validation with Success Guarantee

- untrusted spilling algorithm computes spilling
- iterate over program flow, liveness and spilling information:
  - if free variables of the expression are not in the registers, load them
  - if register set exceeds bound, spill variables
- Properties:
  - yields always a valid spilling
  - if input is a valid spilling it is not modified

References

Matthias Braun and Sebastian Hack. "Register spilling and live-range splitting for SSA-form programs". In: 2009.

Gregory J Chaitin et al. "Register allocation via coloring". In: 1981.

Sebastian Hack, Daniel Grund, and Gerhard Goos. "Towards Register Allocation for Programs in SSA-Form". In: 2005.

Massimiliano Poletto and Vivek Sarkar. "Linear scan register allocation". In: 1999.

Sigurd Schneider, Gert Smolka, and Sebastian Hack. "A Linear First-Order Functional Intermediate Language for Verified Compilers". In: 2015.

Liveness

$$\frac{\mathsf{fv}\, e \subseteq X \quad X_s \setminus \{x\} \subseteq X \quad x \in X_s \quad Z \,|\, \Lambda \vdash \textbf{live}\ s : X_s}{Z \,|\, \Lambda \vdash \textbf{live}\ (\texttt{let}\ x\ \texttt{:=}\ e\ \texttt{in}\ s) : X}\ \textsc{LiveLet}$$

$$\frac{\mathsf{fv}\, e \subseteq X}{Z \,|\, \Lambda \vdash \textbf{live}\ e : X}\ \textsc{LiveReturn}$$

$$\frac{\mathsf{fv}\, e \cup X_{s_1} \cup X_{s_2} \subseteq X \quad Z \,|\, \Lambda \vdash \textbf{live}\ s_1 : X_{s_1} \quad Z \,|\, \Lambda \vdash \textbf{live}\ s_2 : X_{s_2}}{Z \,|\, \Lambda \vdash \textbf{live}\ (\texttt{if}\ e\ \texttt{then}\ s_1\ \texttt{else}\ s_2) : X}\ \textsc{LiveIf}$$

$$\frac{\mathsf{fv}\, \overline{e} \subseteq X \quad \Lambda_f \setminus Z_f \subseteq X}{Z \,|\, \Lambda \vdash \textbf{live}\ f\ \ \overline{e} : X}\ \textsc{LiveApp}$$

$$\frac{X_{s_2} \subseteq X \quad \overline{x} \subseteq X_{s_1} \quad f : \overline{x}; Z \,|\, X_{s_1} :: \Lambda \vdash \textbf{live}\ s_1 : X_{s_1} \quad f : \overline{x}; Z \,|\, X_{s_2} :: \Lambda \vdash \textbf{live}\ s_2 : X_{s_2}}{Z \,|\, \Lambda \vdash \textbf{live}\ (\texttt{fun}\ f\ \overline{x}\ \texttt{:=}\ s_1\ \texttt{in}\ s_2) : X}$$

$$\textsc{LiveFun}$$

Correctness Predicate

$$\frac{L \subseteq M \qquad |R \setminus K \cup L| \leq k \qquad Z \mid \Sigma \mid R \setminus K \cup L \mid M \vdash \mathbf{spill_k}\ \ s : (\emptyset, \emptyset, \_)}{Z \mid \Sigma \mid R \mid M \vdash \mathbf{spill_k}\ \ s : (\emptyset, L, \_)}\ \textsc{SpillLoad}$$

$$\frac{S \subseteq R \qquad Z \mid \Sigma \mid R \mid M \cup S \vdash \mathbf{spill_k}\ \ s : (\emptyset, L, \_)}{Z \mid \Sigma \mid R \mid M \vdash \mathbf{spill_k}\ \ s : (S, L, \_)}\ \textsc{SpillSpill}$$

$$\frac{\mathrm{fv}\, e \subseteq R \qquad |R \setminus K_x \cup \{x\}| \leq k \qquad Z \mid \Sigma \mid R \setminus K_x \cup \{x\} \mid M \vdash \mathbf{spill_k}\ \ s}{Z \mid \Sigma \mid R \mid M \vdash \mathbf{spill_k}\ (\texttt{let}\ x\ \texttt{:=}\ e\ \texttt{in}\ s) : (\emptyset, \emptyset)}\ \textsc{SpillLet}$$

$$\frac{\mathrm{fv}\, e \subseteq R}{Z \mid \Sigma \mid R \mid M \vdash \mathbf{spill_k}\ \ e : (\emptyset, \emptyset)}\ \textsc{SpillReturn}$$

$$\frac{(R_f, M_f) := \Sigma_f \qquad R_f \setminus Z_f \subseteq R \qquad M_f \setminus Z_f \subseteq M \qquad \mathrm{fv}\, \overline{y} \subseteq R \cup M}{Z \mid \Sigma \mid R \mid M \vdash \mathbf{spill_k}\ (f\ \ \overline{y}) : (\emptyset, \emptyset, (R, M))}\ \textsc{SpillApp}$$

$$\frac{|R_f| \leq k \qquad Z \mid \mathrm{merge}\, \Sigma \vdash \mathbf{live}\ s_1 : R_f \cup M_f \qquad f : \overline{x}; Z \mid f : (R_f, M_f); \Sigma \mid R_f \mid M_f \vdash \mathbf{spill_k}\ \ s_1 \qquad f : \overline{x}; Z \mid f : (R_f, M_f); \Sigma \mid R \mid M \vdash \mathbf{spill_k}\ \ s_2}{Z \mid \Sigma \mid R \mid M \vdash \mathbf{spill_k}\ (\texttt{fun}\ f\ \ \overline{x}\ \texttt{:=}\ s_1\ \texttt{in}\ s_2) : (\emptyset, \emptyset, (R_f, M_f))}\ \textsc{SpillFun}$$

$$\mathrm{merge}\ \ \mathrm{nil} := \mathrm{nil}$$

$$\mathrm{merge}\ (R_f, M_f); \Sigma := R_f \cup M_f; \mathrm{merge}\, \Sigma$$

vir-Predicate

$$\frac{x \in \mathcal{V}_R \quad y \in \mathcal{V}_M \quad \textbf{vir}\, s}{\textbf{vir}\, \texttt{let}\ x\ \texttt{:=}\ y\ \texttt{in}\ s}\ \textsc{ViRLoad}$$

$$\frac{\mathrm{fv}\, e \subseteq \mathcal{V}_R \quad \textbf{vir}\, s}{\textbf{vir}\, \texttt{let}\ x\ \texttt{:=}\ e\ \texttt{in}\ s}\ \textsc{ViRLet}$$

$$\frac{\mathrm{fv}\, e \subseteq \mathcal{V}_R}{\textbf{vir}\, e}\ \textsc{ViRReturn}$$

$$\frac{\mathrm{fv}\, e \subseteq \mathcal{V}_R \quad \textbf{vir}\, s \quad \textbf{vir}\, t}{\textbf{vir}\, \texttt{if}\ e\ \texttt{then}\ s\ \texttt{else}\ t}\ \textsc{ViRIf}$$

$$\frac{}{\textbf{vir}\, f\ \overline{y}}\ \textsc{ViRApp}$$

$$\frac{\textbf{vir}\, s \quad \textbf{vir}\, t}{\textbf{vir}\, \texttt{fun}\ f\ \overline{x}\ \texttt{:=}\ s\ \texttt{in}\ t}\ \textsc{ViRFun}$$